# MEASUREMENT OF AIR FLOW

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BY

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LONDON
CHAPMAN & HALL, LTD.
11 HENRIETTA ST., W.C. 2
1927

PRINTED IN GREAT BRITAIN
BY THE ABERDEEN UNIVERSITY PRESS
ABERDEEN, SCOTLAND

# AUTHOR'S PREFACE

DURING the course of his work at the National Physical Laboratory, the author has had to deal with many enquiries relating to air flow measurements. The nature and variety of these questions have convinced him that such measurements frequently present serious practical difficulties to engineers, and this conviction has been strengthened during his personal contact with ventilating engineers and fan manufacturers and users, as the official N.P.L. representative on the Fan Standardisation Committee of the Institution of Heating and Ventilating Engineers. There seems, therefore, to be a real need for a book on the subject, which shall serve both as a text-book for students and as a work of reference for engineers engaged on matters, such as fan engineering and the ventilation of mines and buildings, which involve the measurement of the speeds and pressures of air streams.

So far as the author is aware, no book which adequately meets this need has hitherto been written, and the present work is put forward in the hope that it may prove of value as a first step in this direction. It deals with the theory and technique of the measurement of air flow, the treatment being primarily of a practical nature. No detailed mention is made of methods and instruments suitable only for laboratory use as distinct from industrial conditions, but references to literature dealing with these matters are given, so that those interested in the more specialised aspects of the subject will be able to find the information they require.

The critical survey of relevant published material which the author had to make in preparing this book revealed several important lacunæ in existing knowledge of the subject. Some of these gaps the author has been

able to fill as a result of his own researches, such as that on the design of static tubes, and his work on the theory of the vane anemometer. The former of these researches served to establish the underlying characteristics of static tubes, and to elucidate the features governing their design; whilst the latter led to important practical conclusions relating to the use of vane anemometers.

Although prominence is given throughout to the practical aspects of the subject, no apology is offered for including a certain amount of theoretical matter. Many engineers profess to despise the theorist, but they should reflect that the worker who relies solely on practical experience, whilst he may be adequately equipped as long as his work is entirely of a routine character, will not fail to encounter difficulties when conditions become at all abnormal. These difficulties can only be successfully overcome by a sound knowledge of fundamental principles, and such knowledge cannot be regarded as complete unless it includes the theoretical basis of the methods employed.

The author desires to express his thanks to his colleague, Mr. A. Fage, A.R.C.Sc., for valuable criticism and advice tendered during the writing of the book. He wishes also to make acknowledgment to the Institution of Heating and Ventilating Engineers for permission to reproduce certain material from the paper on the Practical Measurement of Air Flow which he read before that body in October, 1925. Other sources from which the author has reproduced certain of his published work are quoted in the text. Reproduction of Fig. 73, and of part of the subject-matter of Chapter III, including Figs. 6-11, is made by permission of the Controller of H.M. Stationery Office.

E. O.

TEDDINGTON, *August, 1927.*

# CONTENTS

# CHAPTER I.

## INTRODUCTORY.

In many cases where physical measurements have to be conducted a choice of methods is available, and it is for the experimenter to decide which of these is best adapted to his particular purpose. His selection will be governed by considerations of simplicity, directness, and the degree of accuracy he requires; he should always avoid a complicated method when a simpler one will equally serve his purpose. Let us therefore consider what means are available for the problem with which we are here concerned, namely, the measurement of the speeds of streams of liquids or gases, with particular reference to the motion of air along pipes or ducts. A method that immediately commends itself from the standpoint of simplicity is the introduction into the fluid of some indicator such as a small light body, a puff of smoke, or a drop of colouring matter, which will be carried along by the stream and may be timed over a measured distance. Very little consideration is however needed to show that the difficulties and disadvantages of this method are such as to render its practical application inadmissible, except, perhaps, in a few very special instances. The only other direct method that immediately suggests itself is to collect and weigh the quantity of fluid delivered in a given time, whence the mean speed past any section of known area can be calculated if the density of the fluid is known. This method, which is often employed for the measurement of water flow, is simple and convenient in the case of an incompressible fluid whose density is greater than that of the medium surrounding the pipe or channel in which flow is taking place; for gases, the experimental difficulties are serious, and, in the case of air flowing along pipes situated in the atmosphere, so great as to render the method virtually impracticable.

It is not intended in this book to deal with the measurement of the flow of liquids; this question has already been

fully discussed in numerous treatises on hydraulics, and the methods employed have been standardised for some time. We shall here confine attention to the measurement of air flow only, but it should be observed that the methods described will in general be applicable to the flow of other gases with very little, if any, modification, except as regards the constants occurring in the equations to be derived. For our purpose, as we have seen, direct methods are inapplicable, and it is therefore necessary to resort to the measurement of some physical effect arising from the motion. Three such effects have been found by experience to be suitable, namely, pressure changes associated with the motion; mechanical effects, such as the rate of rotation induced in light vanes suitably mounted in the stream; and lastly, the rate of cooling of a hot body, such as an electrically heated wire, introduced into the air current. Of these, the first is of the greatest importance, since, as we shall see later, a properly designed instrument, suitably inserted in the stream, experiences a pressure, commonly called the "velocity head," which is entirely characteristic of the motion, and can be measured on a suitable pressure gauge. If such an instrument is constructed in accordance with certain well-established principles, which are fully explained below, its constant is known and it may therefore be used without calibration as a standard for the measurement of air speed. This is not true of anemometers * which depend for their action either on mechanical or on electrical effects; instruments of both these types are subject to individual variations and require calibration against a standard instrument of the pressure measuring type.

Those instruments of the first class which measure the velocity head are known as pressure-tube anemometers, and are more extensively used in practice than any other type. Other devices which involve the measurement of pressure differences are orifice plates, Venturi tubes, and nozzles. It should be noted, however, that these measure not the velocity head, but pressures which depend on the dimensions and form of the instruments themselves as well as on the motion of the fluid through them. They cannot therefore be used as standards, but require calibration.

Mechanical anemometers comprise three main types—cup, swinging-plate, and vane anemometers. The first two of these

* The word "anemometer," as its derivation implies, is a term which includes all types of instruments used for the measurement of air speed.

are not of importance to engineers, and no attention beyond the following brief description will be devoted to them here. The vane anemometer, on the other hand, which is fully discussed in Chapter VII., is a convenient instrument for practical use and has a comparatively wide field of application.

In its most common form the cup anemometer consists of four hemispherical cups carried, with their bases vertical, at the outer ends of four light arms which are symmetrically disposed in a horizontal plane and are attached to a central sleeve free to rotate about a vertical axis. The cups are arranged in pairs so that the concave side of one member of a pair is presented to the air current at the same time as the convex side of the diametrically opposite cup. At any instant, then, the air on one side of the median plane through the vertical axis of the instrument will be blowing into the interior of one or two cups, and on the other side of this plane on the exterior of the opposite cup or cups. Hence, since the aerodynamic force on a cup with its concave side presented to the wind is greater than when the wind is blowing on its convex face, rotation will ensue at a rate depending on the wind speed. Instruments of this type are bulky and not easily portable; they are chiefly used in meteorological work.

The swinging-plate anemometer consists of a vertical plate, suspended on knife edges, which deflects under the action of a wind to such an angle that the restoring torque due to the weight of the plate is equal to the wind torque. The angle of inclination may therefore be taken as a measure of the air speed. Alternatively, the plate may be mounted on a torsion wire passing through an asymmetric vertical axis; in this case there will be a relation between the wind speed and the amount of twist of the wire necessary to bring the plate into normal presentation to the wind.

Electrical or hot-wire anemometers have not hitherto been very extensively utilised, although certain types have been developed for industrial application. The chief factor militating against their wider use would appear to be the more elaborate apparatus and manipulation that they entail. As laboratory instruments, they may be made to give excellent results, and they are, in particular, well suited to the measurement of low air speeds, a purpose for which they may be designed to have a considerably more open scale than is possible with pressure-tube anemometers.

# CHAPTER II.

## GENERAL PRINCIPLES OF THE PRESSURE–TUBE ANEMOMETER.

### A. Practical Considerations.

Although theoretical analysis is always necessary for the complete elucidation of natural phenomena, it is useful to consider also the practical aspects, in order to enable the physical meaning of the symbols and equations occurring in the analysis to be fully interpreted. For this reason an attempt will be made in the first part of the present chapter to explain from a practical standpoint the principles upon which the action of the pressure-tube anemometer depends. It must not be supposed, however, that the following remarks are intended to convey anything more than a rough mental picture of what occurs. Certain material considerations, which must be included in a complete discussion of the subject, have been neglected for the present, the more theoretical treatment being reserved for a later stage.

The natural starting-point is the consideration of the pressures that exist in a moving fluid, and it is in the conception of these that a good deal of confusion exists amongst practical engineers. For convenience, the pressure of the undisturbed atmosphere—the barometric pressure obtaining at the time and place of measurement—is usually taken as the datum or zero pressure, and all pressures are expressed in terms of the number of units (usually in England inches of water) by which they exceed or fall short of this arbitrary datum. It follows that pressures less than atmospheric will be recorded as negative pressures, but it must be remembered that such pressures are actually positive: the true zero of pressure is, of course, the absolute vacuum, and it is not possible to conceive of a pressure less than this. Except where otherwise stated the customary convention regarding the use of the atmospheric pressure as a datum will be followed in this book, so that it will sometimes be necessary to refer to negative pressures; if the above remarks are borne in mind, however, no confusion will arise.

It is well known that at any point in a stationary fluid a definite hydrostatic pressure exists which acts equally in all directions. If now this fluid is set in uniform, unaccelerated motion, the hydrostatic pressure will still persist although its actual magnitude may be changed, and if we imagine a very small thin plane area placed at any point in the fluid, and moving with it, this " static " pressure, as it is called, will act on the two faces of the plane. Let us assume that there is a continuous stream of the fluid in movement, and that the motion is uniform and does not vary with time, so that at any section perpendicular to the current the conditions are constant. If, instead of moving with the fluid, the small area remains stationary at a point in the section under consideration, and if, moreover, the area is held with its plane parallel to the direction of motion, it is clear that the pressure acting on the two faces will be the static pressure existing at that point in the section of the fluid, provided that the presence of the plane does not disturb the conditions of flow.

Let us now consider what occurs if the area is rotated so that the stream impinges normally on one face ; when the area moves with the stream, it will again be acted upon by the static pressure only, but if it is held stationary at a given point its front face will experience the static pressure and an additional pressure arising from the impact of the moving stream. It is important to observe that this additional impact pressure is entirely characteristic of the motion, depending, as it does, for a plane area of given size, on the speed and density of the fluid only, so that it can be utilised as a measure of the speed.

It appears therefore that if the plane is placed normally to the stream, so that the forward motion of the fluid impinging on it is arrested, the pressure exerted will be the sum of the static and the impact pressures at the point in the fluid at which the plane is situated ; whereas if the plane is placed so that the fluid streams past it without disturbance the static pressure only is experienced. Suppose now that the imaginary plane area is replaced by an open-ended tube of small bore bent to face the stream, and that the other end of the tube is connected to one limb of a simple U-tube pressure gauge, the other limb of which is open to the atmosphere. The forward motion of the fluid impinging on the open end of the tube will again be arrested, and the pressure gauge will record the sum of the impact and static pressures, or the " total head " as it is commonly called (see Fig. 1). It is found by experiment (see below) that

the impact pressure measured by a facing tube is, within wide limits, unaffected by the shape and size of the tube, and is, in fact, equal to the " velocity head " or " velocity pressure " to which reference has already been made in Chapter I. The relation that exists between this pressure and the density and velocity of the fluid will be derived in the second part of the present chapter ; at this stage it is sufficient to observe that the velocity head is a function of the motion only.

We see therefore that whereas the velocity head is uniquely determined by the speed * of the fluid, the static pressure is not ; it follows that the total head in the same fluid may, under two different sets of conditions, be different although the speed in each case may be the same, so that, in order to determine the speed, it is necessary to measure both total head

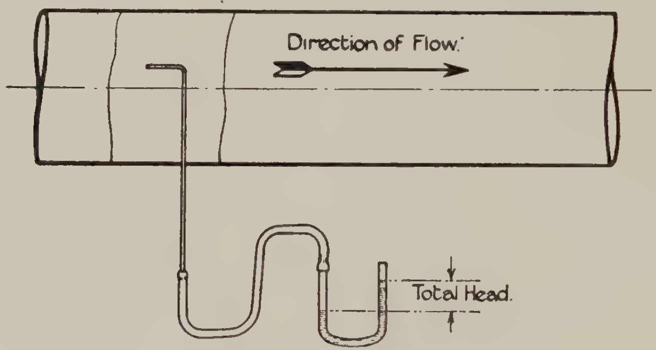



FIG. 1.—Measurement of total head.

and static pressure, since the velocity head cannot be observed separately.

As an example of two instances in which a fluid moves at the same speed but gives different values of the total head, we may consider the case of air which is being moved along a given pipe by means of a fan. It is clear that the air may be made to travel at the same speed whether the fan is blowing or exhausting down the pipe, and the velocity head will be the same in the two cases. The static pressure in the pipe when the fan is blowing, however, will be higher than atmospheric, and lower when the fan is exhausting, so that the total head

* We shall, for simplicity, neglect in this section the effect of variations in density, which is small under conditions usually occurring in air speed measurements.

under the former condition will be higher than that under the latter.

Reverting now to a consideration of the open-ended tube facing an air stream, we see that the pressure gauge to which it is connected will indicate the sum of the velocity and static pressures of the air at the mouth of the tube. If, therefore, the other side of the gauge is connected to a tube or other device which in some manner measures the static pressure, the reading of the gauge will be equal to the difference between the pressures acting on the two sides, i.e. to the velocity pressure, and will thus serve directly to determine the air speed.

It is upon this principle that all pressure-tube anemometers depend. For the measurement of the total head, any form of open-ended tube facing upstream will be reliable, but some care is necessary in the design of the device for measuring the static pressure. The important condition to be observed is that, in the region where the static pressure is measured, the air must not have a velocity component perpendicular to its original direction of motion before the tube was inserted ; in other words, any device for measuring static pressure will give erroneous results if it deflects the air to any appreciable extent. The design of static pressure tubes is fully discussed in the next chapter ; at this stage it is sufficient to note that it is, in fact, possible to make a static tube which measures the static pressure to a degree of accuracy well within practical requirements. Such a tube, suitably used in conjuction with a facing or total head tube constitutes a complete pressure-tube anemometer which does not require calibration, save in exceptional cases, and therefore serves normally as a fundamental standard instrument for the measurement of air speed.

Anemometers of this nature are extensively used in practice. The only disadvantage inherent in this method of measurement is that the velocity pressure corresponding to low air speed is small : a speed of 20 feet per second, for example, only produces a pressure of about 0·09 inch of water. It follows, therefore, that sensitive pressure gauges or "manometers" are necessary for the measurement of low speeds. In fact, for air speeds below about 15 feet per second it is often advisable to use some other measuring device, such as the vane anemometer, the orifice plate or Venturi tube, or, in some cases, the hot-wire anemometer.

## B. Elementary Theoretical Considerations.

In general, the theoretical treatment of problems of fluid motion is extremely complex and demands intricate mathematical analysis, so that only in a few isolated instances of simple motions have theoretical solutions hitherto been found. It is, however, often possible, by the introduction of simplifying assumptions, and with the assistance of experimental research, to deduce relationships, sufficiently accurate for most purposes, between the various properties of the fluid that govern its motion. Such methods have been found to be especially fruitful in the treatment of the flow of liquids and gases in pipes—the particular type of fluid motion with which we are here mainly concerned.

Attempts to visualise conditions of flow are aided by the introduction of the conception of " stream lines." In any mass of moving fluid we assume the field to be mapped out by a system of lines such that the direction of one of these lines at any point coincides with the direction of motion of the particle of fluid situated at the chosen point at the instant under consideration. The stream lines will thus give an instantaneous picture of the flow at any moment. If the flow is unsteady, so that it varies with time, this picture will change from instant to instant ; it may, in the case of periodic motion, undergo a series of changes which recur at regular intervals. For steady flow, on the other hand, the stream lines will preserve their configuration unchanged at all instants, and will show the actual paths of individual particles of the fluid. This type of motion is known as " stream-line " motion, the unsteady variety being termed " turbulent " flow. As we shall see later, both types of flow are possible in pipes, although in practice the latter type occurs by far the more frequently.

A complete mathematical analysis of a case of fluid motion must take into account all the properties of the fluid that influence the flow. It would, however, appear to be justifiable to neglect certain of these properties if the treatment can thereby be appreciably simplified and if, at the same time, it can be shown that the results apply sufficiently closely to actual conditions. Comparison with observation has shown that a certain law, which is of fundamental importance in the flow in pipes, may be simply deduced by neglecting the viscosity of the fluid, i.e. by assuming that no frictional forces occur between adjacent particles of the fluid, or between the fluid

and any solid boundaries with which it is in contact. A fluid of this type is called an inviscid or perfect fluid, and the law in question is named after Bernouilli, by whom it was first stated in 1738. It may be derived in the following manner.

Let ABCD (Fig. 2) represent a longitudinal median section through a tube of fluid bounded by stream lines. (Such a tube is termed a stream tube, and AB and CD are two of the bounding stream lines.) For steady motion, the boundaries of the stream tube will preserve a constant shape, although the actual fluid in the tube will be continually changing.

Consider a small element PQRS of the tube of length $ds$ which is so small that the elements PS and QR of the stream lines may be considered as straight lines. If $v$ and $p$ are respectively the speed and pressure across the section PQ, we may, since $ds$ is small, write $v + dv$ and $p + dp$ for the corre-

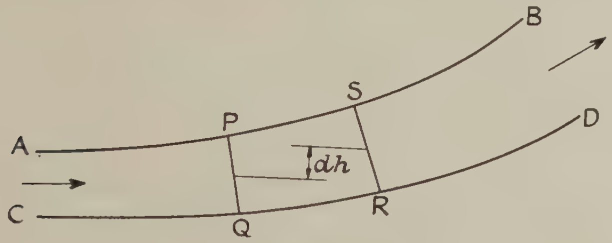



FIG. 2.

sponding quantities across section RS, where $dv$ and $dp$ are small.

Let $a$ = area of section PQ,
$\Delta$ = weight of unit volume of liquid,
$g$ = acceleration due to gravity,
$dh$ = difference of level between the centres of area of PQ and RS.

If second order quantities are neglected, the resultant force along the axis of the tube due to the pressure on the faces of the element can be shown to be equal to $adp$, acting in the direction opposite to that of the motion. To this must be added the force acting in the same direction due to the weight of the fluid in the element PQRS. The latter force is equal to $\Delta adh$ acting on PQ and is zero on RS, the resultant being $\Delta adh$. The sum of these forces is equal to the rate of change of momentum through the element. The mass of fluid entering per second

across PQ is given by $\frac{\varDelta}{g}\,av$, and, since PS and QR are stream lines, the mass leaving at RS is also $\frac{\varDelta}{g}av$, so that the change of momentum per second is $\frac{\varDelta}{g}\,avdv$, in the direction of motion.

We therefore have the equation

$$\frac{\varDelta}{g}avdv = -\,adp - \varDelta adh,$$

or

$$\frac{\varDelta}{g}vdv + dp + \varDelta dh = 0. \qquad \text{(1)}$$

The quantity $\frac{\varDelta}{g}$ is usually denoted by $\rho$, so that (1) may be written

$$vdv + \frac{dp}{\rho} + gdh = 0.$$

On integration this equation becomes

$$\frac{v^2}{2} + gh + \int\frac{dp}{\rho} = \text{constant} = \text{C}, \qquad \text{(2)}$$

which applies to all sections of a given stream tube. The quantity $h$ may be taken as the height of the section measured above any arbitrary datum. An alteration in the datum level will merely involve an alteration in the value of C for the stream tube. When the stream tube becomes infinitely narrow it may be considered as forming a stream line, and we may therefore say that equation (2) gives the relation between pressure and velocity along a stream line.

If the flow is horizontal, the datum level may be taken as being that of the stream line itself, and $h$ becomes zero. In practice, when dealing with the flow of air it is almost invariably found that the quantity $gh$ is negligibly small in comparison with the sum of the other two terms of equation (2), so that the simplified form of this equation, viz.,

$$\frac{v^2}{2} + \int\frac{dp}{\rho} = \text{C}, \qquad \text{(3)}$$

which is strictly only applicable to the case of horizontal flow, may generally be used without appreciable error. In the

following treatment we shall therefore not consider differences of level.

A further simplification can generally be made, namely, to treat the fluid as incompressible, so that $\rho$ is constant and (2) becomes

$$\tfrac{1}{2}\rho v^2 + p = C, \qquad . \quad . \quad . \quad (4)$$

which is Bernouilli's equation in the form most commonly used in practical work. For air, the effect of compressibility can be neglected up to speeds of about 200 feet per second, so that (4) holds for all conditions ordinarily encountered.

On consideration it will be seen that both terms occurring on the left-hand side of (4) have the dimensions of force per unit area, i.e. of a pressure; the first term is the quantity we have called the velocity head, the second is the static pressure, and the sum of the two is the total head. Bernouilli's equation is therefore equivalent to the statement that the total head along a stream line is constant. This equation has been derived on the assumption that the flow is frictionless, but it can be shown that it holds also for the stream line flow of a viscous fluid. Further, experiments indicate that it can be applied with good accuracy in many cases of turbulent flow of actual fluids.

We may now consider the behaviour of the facing tube in the light of Bernouilli's equation. If a stream line is brought to rest by causing it to impinge on a small obstacle introduced into the fluid, the pressure $p_1$ acting on the obstacle will, if the latter be assumed to be so small as to affect the one stream line only, be equal to the total head at that point in the free stream before the obstacle was introduced, i.e.,

$$p_1 = \tfrac{1}{2}\rho v_0^2 + p_0, \qquad . \quad . \quad . \quad (5)$$

where $v_0$ and $p_0$ are respectively the velocity and static pressure at the point in the unobstructed stream.

This pressure $p_1$ will be the pressure acting at the mouth of an open ended tube facing the stream. Theoretically this is only true if (*a*) the tube is so narrow that one stream line only is brought to rest, (*b*) flow through the tube is prevented, as it is if the other end is connected to a manometer, (*c*) the effects of viscosity on the action of the tube can be neglected, and (*d*) the tube does not disturb the flow. Of these, condition (*b*) is the only one which can be definitely fulfilled, and the fact that a tube of finite size facing the air current does

actually measure the total head rests not upon a rational theoretical basis but upon a large amount of carefully obtained experimental evidence. Such experiments show beyond doubt that the total head is accurately measured by a facing tube even in turbulent flow, and further that, provided it has an orifice facing upstream, variations in the size and shape of the tube within wide limits are not accompanied by any errors in the readings. As regards the effect of viscosity, experiments by Miss Barker * have demonstrated that this can be neglected except for very small tubes working at very low speeds, outside the range ordinarily encountered in practice.† It seems therefore that the facing tube will give an accurate reading of $p_1$ under practical conditions, although theoretically this result could only be expected for the stream-line flow of a perfect inviscid fluid with an infinitely small tube.

Now we see from (5) that $p_1$, the total head at the point occupied by the mouth of the facing tube, is equal to the sum of the velocity head and the static pressure in the undisturbed flow at that point. If therefore $p_0$ is measured by a properly designed static pressure tube connected to the other side of the same manometer to which the total head tube is connected, the resultant pressure R indicated by the manometer is given by

$$R = p_1 - p_0 = \tfrac{1}{2}\rho v_0^2, \qquad (6)$$

from which $v_0$ can be calculated. Details of the method of calculation are given in Chapter V.

**Reading of a Total Head Tube at High Speeds.**—The equation just derived for the pressure at the mouth of a total head tube is strictly valid only when the fluid is incompressible, but we shall see immediately that it applies with sufficient accuracy for most practical cases of air flow. The general equation which takes account of compressibility, that is, changes of density with pressure, may be obtained as follows.

Before the total head tube is inserted into the stream the pressure and velocity are related according to equation (3), viz.,

$$\frac{v_0^2}{2} + \int \frac{dp_0}{\rho_0} = C. \qquad (7)$$

* *Proc. Roy. Soc.*, 1922, vol. 101, A., p. 435.

† For air at ordinary temperatures and pressures, viscosity effects on the total head tube may be neglected, provided that the product $vd$ is greater than 0·01, where $v$ is the air speed in feet per second and $d$ the diameter of the tube in feet.

When the tube is inserted we assume that the velocity at the mouth is reduced to zero and the pressure becomes $p_1$, so that, by application of Bernouilli's theorem,

$$\int \frac{dp_1}{\rho_1} = \mathrm{C}. \quad . \quad . \quad . \quad . \quad (8)$$

We may now equate the left-hand sides of (7) and (8) and thus obtain

$$\frac{v_0^2}{2} + \int \frac{dp_0}{\rho_0} = \int \frac{dp_1}{\rho_1}. \quad . \quad . \quad . \quad (9)$$

The assumption, which appears to be justified by experimental evidence, is now made that pressure changes are sufficiently rapid to be adiabatic, so that $\frac{p_1}{\rho_1^\gamma} = \frac{p_0}{\rho_0^\gamma} = \mathrm{K}$, where $\gamma$ for dry air has the value 1·408.

Hence, from (9)

$$\frac{v_0^2}{2} + \mathrm{K}^{\frac{1}{\gamma}} \int \frac{dp_0}{p_0^{\frac{1}{\gamma}}} = \mathrm{K}^{\frac{1}{\gamma}} \int \frac{dp_1}{p_1^{\frac{1}{\gamma}}},$$

which, on integration and substitution of $\left(\frac{p_0}{\rho_0^{\frac{1}{\gamma}}}\right)$ for K, reduces to

$$\frac{p_1}{p_0} = \left[1 + \frac{\gamma - 1}{\gamma} \cdot \frac{\rho_0}{p_0} \cdot \frac{v_0^2}{2}\right]^{\frac{\gamma}{\gamma - 1}}.$$

Expanding by the binominal theorem we obtain

$$\frac{p_1}{p_0} = 1 + \frac{\rho_0}{p_0}\frac{v_0^2}{2} + \frac{1}{\gamma}\frac{\rho_0^2}{p_0^2} \cdot \frac{v_0^4}{8} + \frac{2 - \gamma}{\gamma^2} \cdot \frac{\rho_0^3}{p_0^3}\frac{v_0^6}{48} + \ . \ . \ .,$$

or

$$p_1 = p_0 + \frac{\rho_0 v_0^2}{2}\left[1 + \frac{\rho_0}{\gamma p_0} \cdot \frac{v_0^2}{4} + \frac{(2 - \gamma)\rho_0^2}{\gamma^2 p_0^2} \cdot \frac{v_0^4}{24} + \ . \ . \ .\right]. \quad (10)$$

Now $a_0$, the velocity of sound in air at pressure $p_0$ and density $\rho_0$, is equal to $\sqrt{\frac{\gamma p_0}{\rho_0}}$.

Hence (10) may be written

$$p_1 = p_0 + \frac{\rho_0 v_0^2}{2}\left[1 + \frac{1}{4}\frac{v_0^2}{a_0^2} + \frac{2 - \gamma}{24} \cdot \frac{v_0^4}{a_0^4} + \ . \ . \ . \ \right], \quad (11)$$

which is the general expression for the pressure at the mouth of a total head tube.

For air at ordinary temperatures and pressures $a_0$ may be taken as roughly 1100 feet per second, and it is then easy to see from (11) that $v_0$ may be as high as 200 feet per second before the second term in the expansion becomes equal to 0·01. Hence for all speeds below this the second and subsequent terms may be neglected and the ordinary form of (11), viz.,

$$p_1 = p_0 + \tfrac{1}{2}\rho_0 v_0^2,$$

may be employed. For very high speeds, exceeding the velocity of sound, Rayleigh * and Stanton † have shown that (11) requires modification on account of the effects of stationary waves, but this point is outside the scope of the present work.

* *Proc. Roy. Soc.*, A., 1910, vol. 84. † *Ibid.*, 1926, vol. 111.

# CHAPTER III.

## THE DESIGN OF PITOT AND STATIC TUBES.

It was shown in the last chapter that the velocity head is, in general, obtained as a difference of two pressure observations—the total head and the static pressure—of which the former can be accurately determined by means of an open-ended tube facing the stream. This method is, in fact, universally adopted for the measurement of total head, the facing tube being known as the Pitot tube after the French scientist Pitot, who appears to have been the first to use it for this purpose. Abundant experimental evidence exists in confirmation of the fact that the pressure indicated by the facing tube is accurately equal to the total head, and that the form of the tube may be varied within wide limits without sensibly affecting the observed pressure. No such latitude is possible in the design of a tube to measure static pressure: for this purpose it is essential to observe the condition already stated, namely, that the flow past the static orifice must be, as far as possible, the same, both in speed and direction, as it was in the free stream undisturbed by the presence of the instrument.

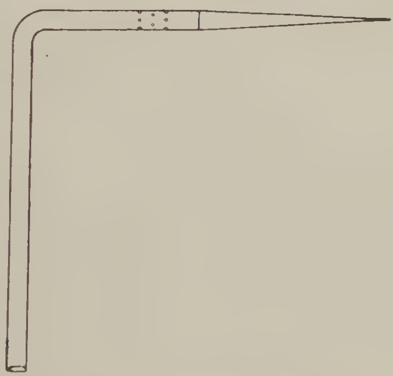


Fig. 3.—Static tube.

On the basis of these requirements the development of static pressure heads has proceeded upon the general lines of the instrument shown in Fig. 3. It will be seen that the head of the tube which is introduced into the air current is bent at right angles to face the stream, its walls being parallel to the direction of flow, and terminates in a conical plug of gradual taper merging smoothly into the tube without a gap or change

in external diameter. This form of head deflects the air only slightly, and at a short distance back from the tapered portion, where the flow is once more sensibly parallel to the tube, a number of small holes is drilled. The other end of the tube is connected to a manometer, and if the head is suitably proportioned (see below) the true static pressure will be observed. In drilling the holes in the walls of the tube it is essential that no burrs be left on the outside.

**The Pitot-Static Tube.**—Fig. 4 shows a convenient form

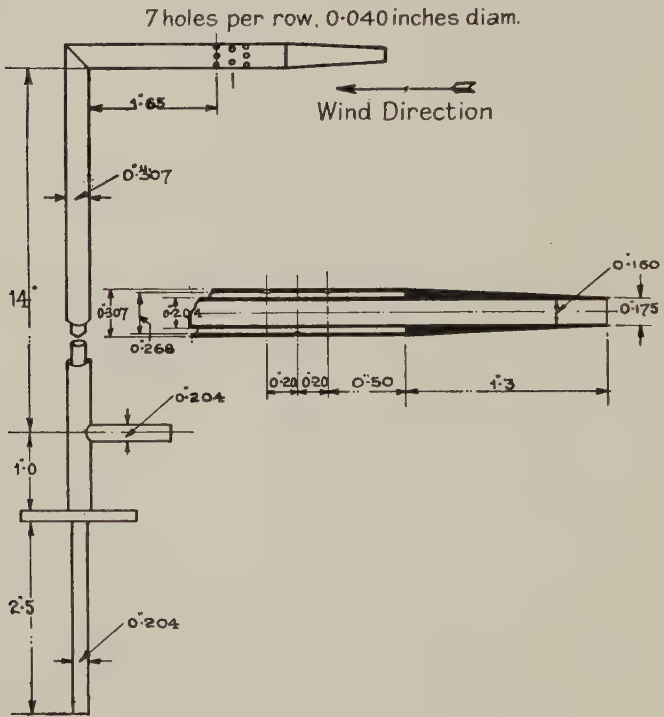



FIG. 4.—N.P.L. Standard Pitot-static tube.

of combined Pitot and static instrument in which the two tubes are arranged concentrically. The apparatus is constructed of brass tubing to the dimensions shown in the diagram, the outer tube, which serves for the determination of static pressure, being tapered and closed at the end, with the static holes arranged on the parallel portion, while the inner tube measures the total head. This instrument was devised and calibrated

at the National Physical Laboratory,* and the static side was found to indicate the true static pressure to an accuracy of 0·1 per cent., which was within the limits of observational error. In order to effect a measurement of air speed, the two tubes are connected by means of rubber tubing to opposite sides of a differential manometer,† and the resultant reading indicates the speed of the air past the mouth of the tube facing the current. It should be noticed that the static pressure is measured about 2 inches downstream of the region at which the total head is observed. Strictly, of course, the two pressures should be measured at the same point if the speed is to be deduced from the readings. No sensible error is, however, introduced by the more convenient practical arrangement adopted, since the static pressure gradient along an airstream is normally never sufficiently great to give rise to an appreciable difference of static pressure at two points so near together.

In view of the satisfactory performance of this instrument, it has been adopted by the National Physical Laboratory as their fundamental standard against which all other anemometers are calibrated. Copies of this standard are now in general use, and it may be noted that these copies do not require calibration if they conform faithfully to the dimensions of the standard shown in Fig. 4, particularly as regards the taper of the head, and the distances back from the nose of the static holes and the stem (i.e. the portion of the instrument perpendicular to the head). The static pressure reading is sensitive to quite small variations in these dimensions, and if for some reason it is convenient to construct the instrument of tubing of diameter different from that shown in the diagram, it is essential to alter the external dimensions of the head in the ratio of the new to the original diameter, so that the new head is geometrically similar to its prototype.

Although this instrument provides a perfectly definite and trustworthy standard, it is subject to certain defects when considered from the standpoint of the practical engineer. Reference has been made to the need for accurate reproduction of the shape of the head in the manufacture of duplicates, if the calibration is to remain unchanged, and it will be instructive to examine more closely the reason for this necessity. The

* Reports and Memoranda, No. 71, of the Aeronautical Research Committee—December, 1912.

† See Chapter IX.

pressure distribution in the vicinity of the nose of the instrument has never been actually determined, but there can be little doubt that it will be somewhat of the nature of that shown in Fig. 5, in which the ordinates represent the pressures normal to the surface of a model of an airship envelope. It will be seen that near the forward end there is a region of positive pressure (i.e. above atmospheric) which is rapidly succeeded towards the tail by a negative pressure zone extending for a considerably greater length. Experience indicates that the pressure distribution round the nose of the static tube is similar to that shown in Fig. 5 with, probably, the forward region of positive pressure relatively much shorter, and that the static holes, as normally situated, lie within the low pres-

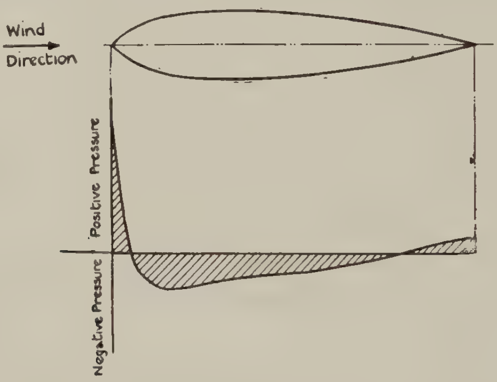



FIG. 5.—Distribution of pressure on airship hull.

sure area, so that, in the absence of other effects, the static pressure recorded will be too low. It is found also, however, that the presence of the stem introduces a region of higher pressure upstream of itself which extends to a distance well beyond the position of the static holes.

There is thus no *a priori* reason why the pressure recorded at the static holes in the standard static head should be the actual static pressure; the fact that it is so would appear to be fortuitous, and to depend upon the somewhat precarious balancing of opposite effects—a positive pressure due to the stem and a suction due to the flow round the nose. This fact assumes importance when, as comparatively often happens in practice, it is convenient to employ a long, straight form of instrument in which the stem is either absent, or considerably further down wind from the nose than in the standard form.

In such circumstances the observed static pressure will be in error, even if the head itself is otherwise of standard shape. It would obviously be an advantage to have as a standard a form of head whose readings would remain sensibly constant irrespective of the position of the stem and, within reasonable limits, of the shape of the nose.

**The Characteristics of Static Tubes.**—In order to establish the characteristics of static tubes and to determine the chief factors influencing their design, the author, in conjunction with Mr. F. C. Johansen, B.Sc., undertook a research on the subject which is described in Reports and Memoranda, No. 981,* of the Aeronautical Research Committee. The investigation was designed to throw light on the best position for the static holes, the effect of variations in the shape of the nose, and the influence of the stem on the static readings. For this purpose a straight brass tube $\frac{5}{8}$ inch in external diameter and about 5 feet long was selected. At one end provision was made for fitting a rubber connecting tube leading to a sensitive manometer; the other end was faced square and tapped internally to take any one of a number of differently shaped heads, which were similarly screwed and were thus interchangeable. Starting from this end, twenty sections were chosen along the tube, and at each section four pressure holes, 0·05 inch in diameter, equally distributed round the circumference, were drilled through the walls of the tube. The spacing of the sections and details of the tube and the heads are shown in Fig. 6.

Four heads in all were tested; of these, three were tapered, one, which had a taper of one in ten, being geometrically similar to the head of the N.P.L. standard instrument (see Fig. 4), and the other two having respectively half and twice this taper. In each case the maximum outside diameter was twice the diameter at the smaller end of the taper, and the head was bored out internally for some distance back from the nose, as shown in Fig. 6, in order to represent the front of the impact tube. The fourth head was hemispherical, the diameter of the aperture representing the impact tube being, as for the other heads, half the diameter of the main tube. A narrow shoulder turned on the base of each head, as shown in the diagram, fitted against the faced head of the main tube, and served to provide a flush joint which was easily rendered leak-tight by the application of a small quantity of vaseline to the screw thread.

* *The Design of Pitot-Static Tubes* (Ower and Johansen). See also *Colliery Engineering*, October, 1927.

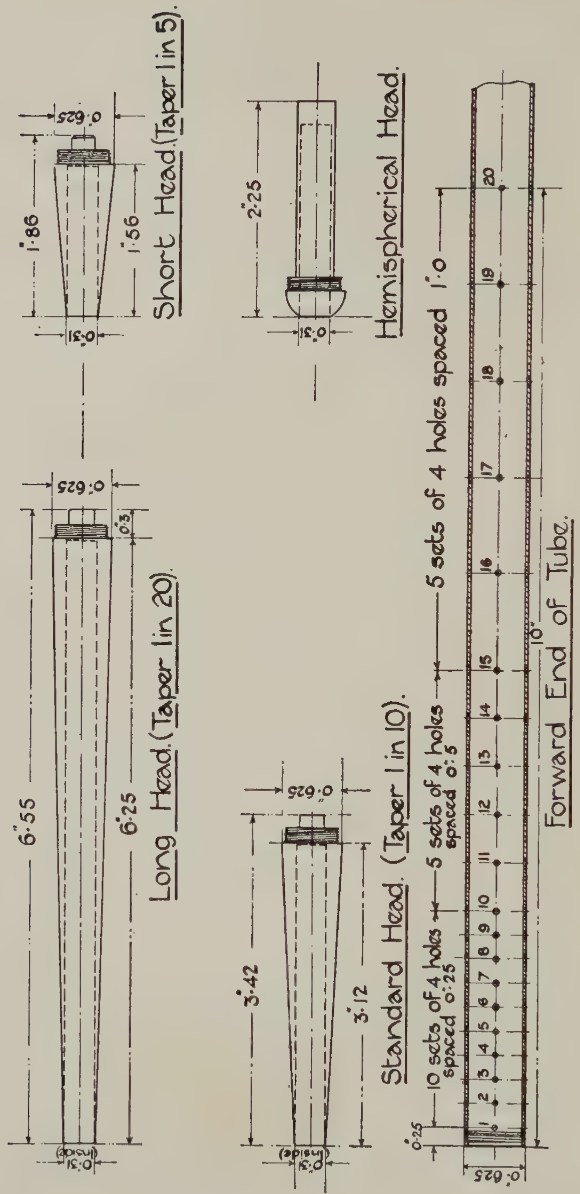

Short Head. (Taper 1 in 5).
Hemispherical Head.
Long Head. (Taper 1 in 20).
Standard Head. (Taper 1 in 10).
Forward End of Tube.
5 sets of 4 holes spaced 1"·0
5 sets of 4 holes spaced 0"·5
10 sets of 4 holes spaced 0"·25
1"·86
1"·56
0"·625
0"·31
2"·25
6"·55
6"·25
0"·3
3"·42
3"·12
0"·31 (inside)
0"·25
10"

Fig. 6.

Some doubt was at first felt as to the requisite depth of the openings representing the mouth of the impact tube. It was thought that this depth might conceivably affect the flow round the nose, and thus also the pressures observed at the various static holes. Preliminary experiments with two of the heads, however, in which the depth was varied by means of cylindrical plugs fitting the opening, showed that no measureable effect due to this cause could be detected, even when the forward end of the plug was flush with the mouth of the opening.

For full details of the experiments the reader is referred to the original report mentioned above. Briefly, the investigation consisted in supporting the tube, with one or other of the heads attached, with its length along the axis of a wind tunnel,* i.e. parallel to the wind, and observing the difference between the pressure recorded at the pressure holes at each section in turn, and the true static pressure in the empty tunnel as measured by the N.P.L. standard instrument. Initially, all the pressure holes were stopped up with a mixture containing equal parts of ceresin and vaseline previously melted up together, which formed a conveniently plastic medium for making the holes leak-tight. The surplus was then cleaned off, leaving a flush smooth outer surface to the tube, and the pressure holes at the section at which measurements were to be made were opened, so that the pressure acting on the tube at this section was transmitted to the manometer. When this pressure had been observed the holes were again closed with the filling medium, and the procedure was repeated for the next set of holes, and so on until the pressure at each section had been obtained and compared with the true static pressure at the position occupied by the section. Observations were made at a number of air speeds up to a maximum of 70 feet per second, with the tube fitted in turn with each of the four heads. The data furnished by these experiments, which we may for convenience term series (*a*), served to establish the effect of variations in the shape of the nose, and to indicate the best position of the static holes. For the determination of the effect of the presence of the stem (series (*b*)) measurements were made at

* A wind tunnel is a large duct of square or circular cross-section along which a uniform steady current of air is made to move at a known speed, which can be adjusted at will to any value within certain limits. It is a piece of apparatus much used in aerodynamic research. For a full description reference may be made to *Applied Aerodynamics* (Bairstow), or to the reports of the Aeronautical Research Committee.

one set of holes only, the pressures there being recorded for various positions of a vertical rod representing the stem of the combined Pitot and static instrument and of the same diameter as the main tube, as the rod was moved along the tube towards the holes, starting from a considerable distance downstream. The holes selected for these measurements were situated in the twelfth section from the forward end of the tube, since the results of series (*a*) indicated that for all the heads tested the pressure variation along the tube had become inappreciable at this section.

The results of these experiments supplied comprehensive information relating to the design of static tubes, and they are discussed in some detail below.

**Series** (*a*).—Fig. 7 shows data obtained with the four heads at an air speed of 70 feet per second. The ordinates represent the differences between the observed pressures at the various sections and the true static pressure at these sections, the differences being expressed as percentages of the quantity $\frac{1}{2}\rho v^2$; the abscissæ denote the distances of the sections from the forward end of the main tube (i.e. from the commencement of the parallel portion downstream of the head) expressed in terms of the external tube diameter. It should be noted that, expressed in this manner, any selected ordinate represents the percentage error on $\frac{1}{2}\rho v^2$ which would be obtained from a total head tube used in conjunction with a static tube of the form shown and having its pressure holes in the selected position.

It will be seen from Fig. 7 that in every case a lower pressure was recorded than that observed with the standard instrument, and that the gradient of normal pressure along the tube exhibited the same characteristics for all the heads tested. For the first 3 or 4 tube diameters from the commencement of the parallel portion the pressure variation is rapid, but from about 5 diameters onwards the mean gradient becomes almost zero, and the pressure tends to approach a constant value. A noteworthy feature of these measurements is the fact that the position of the beginning of the parallel portion of the tube appears to be the main factor which determines the longitudinal pressure gradient. Thus, the longest head (taper 1 in 20) extended 10 diameters upstream of the end of the tube to which it was attached, and the shortest—the hemispherical head—only about $\frac{1}{2}$ diameter, a difference of $9\frac{1}{2}$ diameters. Nevertheless, the difference in the positions at which the gradients for these two heads become sensibly horizontal is only about

3 diameters, the region of approximately constant pressure being reached at about 3 diameters with the long head, and with the hemispherical head at about 6 diameters, from the end of the main tube. It should also be observed that the ordinates of the more or less horizontal portion of the gradient are approximately the same for all the four heads, at any rate up to

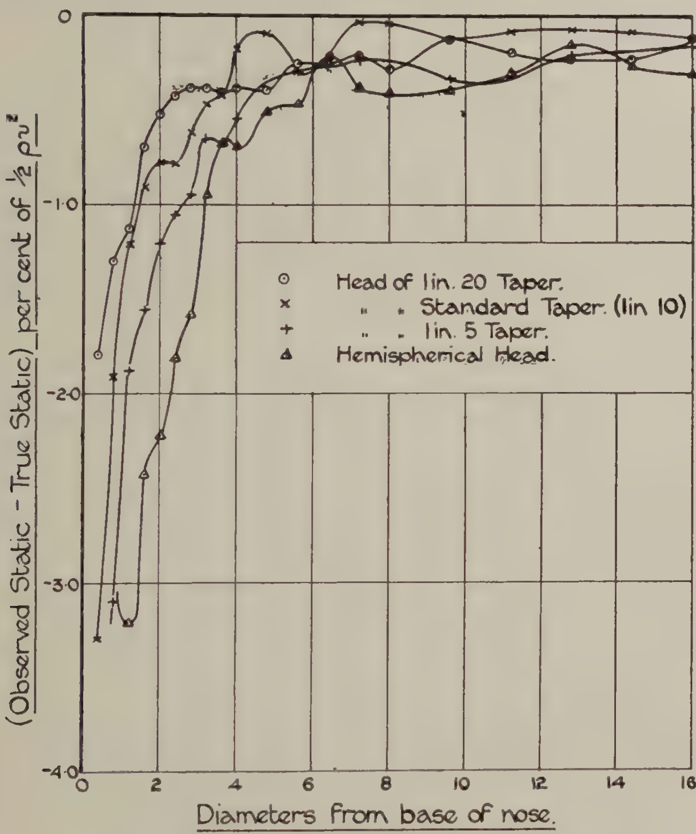



FIG. 7.

20 diameters from the end, which represented the limit of the experiments. A mean value for this ordinate would appear to be about — 0·25 per cent. of $\frac{1}{2}\rho v^2$.

The effect of the shape of the head is clearly shown by the results as plotted. Over the region where the pressure gradient is steep, the ordinates at any position along the tube are in

the order of bluffness of the four heads, being greatest for the shortest and bluffest head (hemispherical) and least for the longest and most gradual head—that of 1 in 20 taper. After the pressure has become approximately constant along the tube, however, no systematic effect due to the shape of the head can be detected.

It is possible at this stage to consider, in the light of these experiments, where the static holes can best be placed. Reference to Fig. 7 indicates that it is inexpedient to put them at a distance less than 5 diameters from the end of the tube, since, in addition to the steepness of the pressure gradient, which is itself an undesirable feature if steadiness and constancy of readings are to be obtained, the variation with the shape of the head in this region is pronounced. In the existing N.P.L. standard the centre row of static holes is only 2·3 diameters from the commencement of the parallel length of the tube, and it will be evident that in these circumstances the apparatus, regarded as a precision instrument, might be expected to be unduly susceptible to relatively small changes in the shape of the head. It would appear, however, in view of the steep pressure gradient at this position, that a more effective cause of small differences in calibration from one instrument to another would be small errors in the longitudinal disposition of the static holes. In a head which would permit of more latitude in manufacture without affecting its calibration, the static holes should be about six diameters back from the end of the tube, where the pressure gradient is gradual and the effect of an alteration in shape is, within very wide limits, inappreciable.

**Series** (*b*).—The effect of the stem was observed for three heads, and the results are reproduced in Fig. 8. In this diagram the abscissæ represent the distances of the stem aft of the static holes (in this case those situated in the twelfth section from the forward end of the tube) expressed in terms of the tube diameter. The ordinate at any abscissa represents the excess pressure produced by the stem in this position over the pressure at the section when the stem is absent, this excess pressure being, as before, expressed as a percentage of $\frac{1}{2}\rho v^2$. It will be seen that the presence of the stem increases the pressure in front of it, the effect being appreciable for a considerable distance upstream ; a further feature which attracts attention is the fact that the shape of the head exerts no perceptible influence on the observed increase of pressure. This increase

is very marked at a short distance ahead of the stem ; when the latter is 2 diameters behind the static holes, for example, the pressure is raised by nearly 4 per cent. of $\frac{1}{2}\rho v^2$, and the

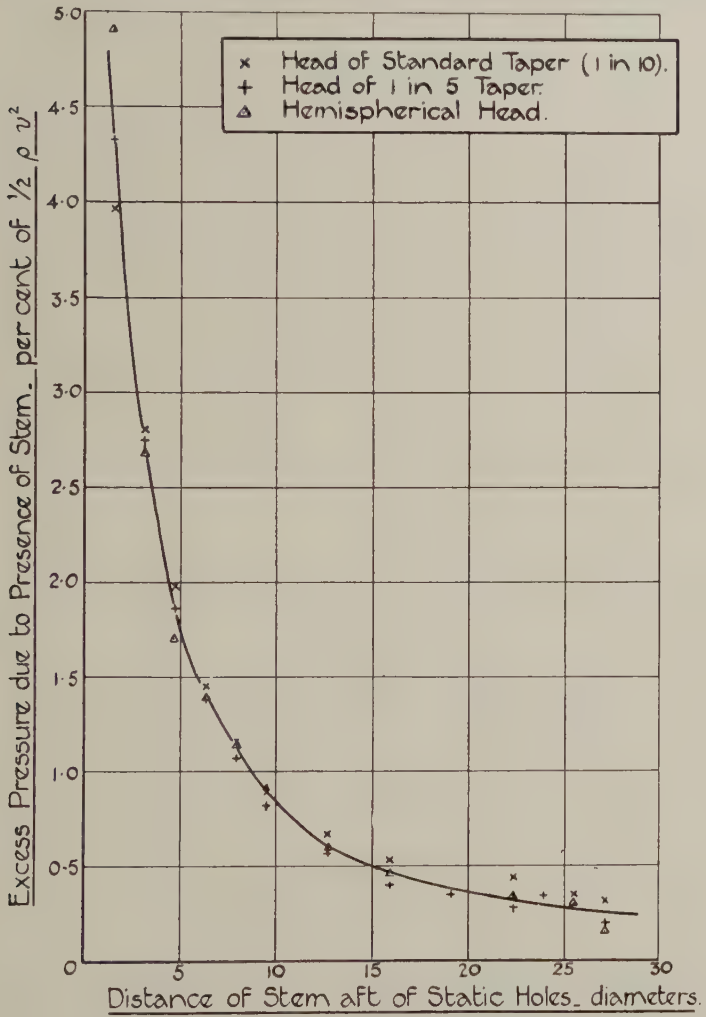



FIG. 8.—Effect of stem on pressure at static orifices.

increase does not fall to 0·5 per cent. until the stem is 15 diameters behind the holes.

In the N.P.L. standard instrument the stem is some

$6\frac{1}{2}$ diameters behind the median plane of the static holes. Reference to Fig. 8 shows that, at this distance, the presence of the stem augments the static pressure by about 1·4 per cent. If an instrument is used, therefore, which conforms in all respects to the N.P.L. standard, except that the stem is absent, it is to be expected that an error of 1·4 per cent. on $\frac{1}{2}\rho v^2$, i.e. 0·7 per cent. on $v$, will be incurred, which is unduly large for accurate work.

Definite indications are furnished by the results of experiments (*a*) and (*b*) of the features to be observed in the proportioning of static heads. From series (*a*) it is apparent that the static holes should be not less than 6 diameters back along the parallel portion of the tube, and from experiments (*b*) that the stem should be at least 15 diameters behind the static holes.* These two requirements would combine to produce a head which would be considerably longer than that of the existing standard instrument, but this unavoidable increase in length can be appreciably reduced by substituting a hemispherical nose for the present taper. This course would appear to be permissible, since the results of experiments (*a*) showed that the effect of the shape of the nose was not felt beyond about 6 diameters downstream from the end of the parallel portion. By the use of such a hemispherical nose a saving in length of $4\frac{1}{2}$ diameters will be effected, and there will be the additional advantage that the distance between the static holes and the mouth of the total head tube will not be so great that the effect of a static pressure gradient along the direction of the airstream will be felt. In fact, in the proposed arrangement, with tubes of the same diameter as those at present used in the N.P.L. standard, this distance will be slightly less than the corresponding distance on the standard.

This reasoning led to the construction of the modified Pitot-static tube shown in Fig. 9. The tubes used were of the same diameter as those of the original standard, the outer one being 0·307 inch external diameter, and the mouth of the total head tube 0·157 inch inside diameter. The single row of 7 static holes, 0·038 inch in diameter, was situated 6 external diameters back from the base of the hemispherical nose, and the axis of the stem was 15 diameters behind the holes, so that the total length of the head was 6·58 inches.

* Observance of the latter condition ensures that the error in static pressure incurred by the use of an instrument in which the stem is entirely absent will be limited to 0·5 per cent. of $\frac{1}{2}\rho v^2$.

The static side of this instrument was calibrated in the wind tunnel against the N.P.L. standard tube, at wind speeds ranging from 20 to 70 feet per second, and it was found that the modified tube indicated a slightly lower value for the static pressure than the standard, the mean difference over the range of wind

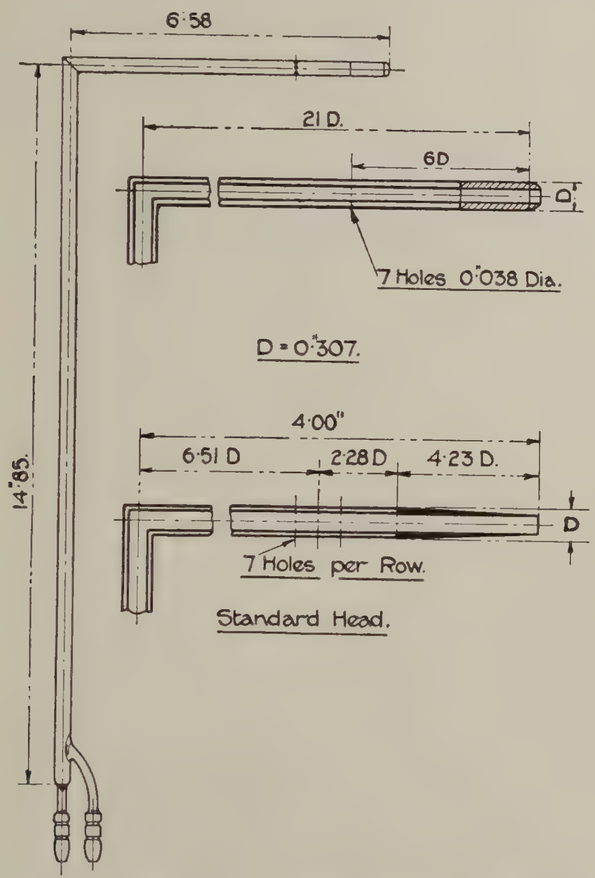



FIG. 9.—Pitot-static tube with hemispherical head.

speeds tested amounting to 0·6 per cent. of $\frac{1}{2}\rho v^2$. On the assumption that the total head tube reads accurately (and one or two tests indicated that this was the case), it is to be inferred, therefore, that the readings of the combined instrument will be 0·6 per cent. of $\frac{1}{2}\rho v^2$ higher than those given by the existing standard.

It may be observed that by putting the stem further forward the error in the static pressure recorded by this instrument can be entirely eliminated. For this purpose the stem must be advanced so that it produces, at the region occupied by the static holes, a pressure which is 0·6 per cent. of $\frac{1}{2}\rho v^2$ greater than that in the instrument as actually designed. Thus, instead of giving a pressure of 0·5 per cent. of $\frac{1}{2}\rho v^2$ at the holes, the stem must be situated so that it gives a pressure of 1·1 per cent. of $\frac{1}{2}\rho v^2$, and Fig. 8 shows that this occurs when the stem is about 8 diameters behind the holes. An instrument having a head shaped as in Fig. 9, with the stem 8 diameters behind the static holes, may be relied upon to measure wind speed to an accuracy of about $\pm$ 0·1 per cent.

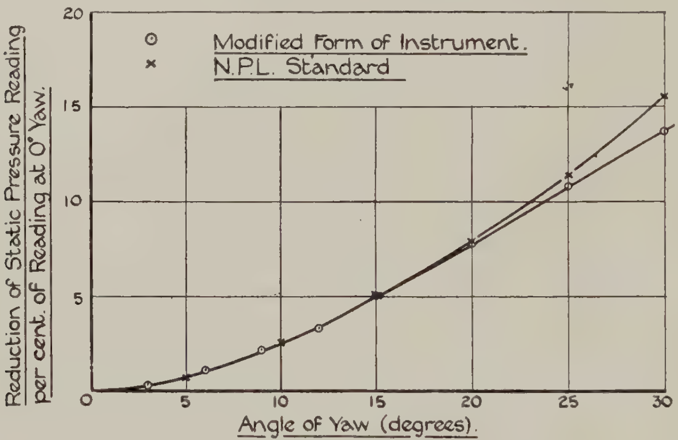



FIG. 10.—Effect of yaw on static pressure reading.

Tests were also conducted with the modified instrument to determine the effect of inclinations of the head to the wind direction. For this purpose the instrument was mounted with the stem vertical in such a manner that the head could be rotated or "yawed" through known angles in a horizontal plane. The results are plotted in Figs. 10 and 11, which relate respectively to the effect of yaw on the static pressure readings, and on the readings of the combined instrument, i.e. the square of the velocity head. For static pressure the results are given as percentages of the reading at zero angle of yaw (head parallel to wind), and it will be seen that the effect of yaw was to reduce the pressure at the holes. As regards the readings of the combined instrument, the results are expressed in the form of the

ratio $\frac{P_\theta}{P_0}$, where $P_\theta$ and $P_0$ are the readings at $\theta°$ and 0° yaw respectively. The results show that this ratio increased up to 16° yaw, where it had a maximum value of 1·045, and then decreased, passing through unity at about 25·5°. It appears therefore that, up to this angle, the maximum error that will arise in the measurement of velocity with this instrument will be + 4·5 per cent. of $v^2$, i.e. 2·3 per cent. of $v$; up to 30° yaw the error in $v$ may be ± 2·3 per cent., and the instrument may be as much as 11° out of alignment before the error exceeds 1 per cent. For comparison, the results of similar tests on the

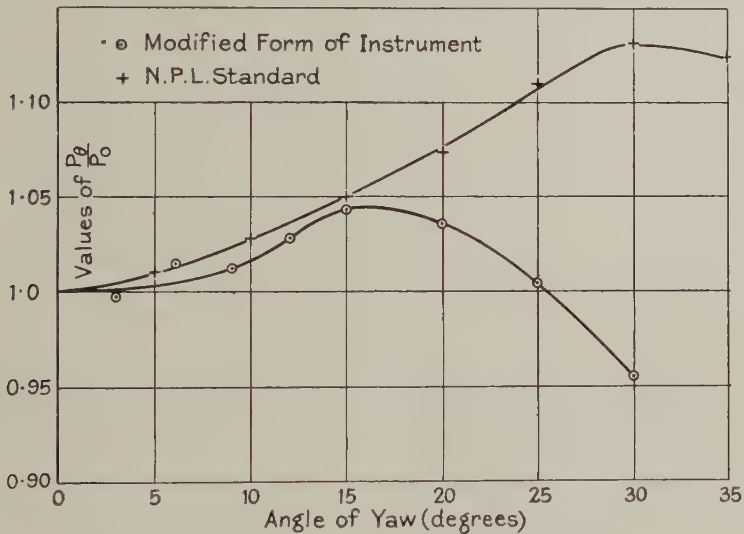



FIG. 11.—Effect of yaw on velocity head reading.

N.P.L. standard instrument are included in Figs. 10 and 11, and indicate the definite superiority of the modified instrument as regards sensitivity to errors of alignment to the true wind direction. For example, between 0° and 30° yaw the possible error in the measurement of air speed with the standard is about $5\frac{1}{2}$ per cent. as compared with the corresponding maximum of 2·3 per cent. with the modified type.

In view of the satisfactory performance of this instrument the author is definitely of the opinion that it may with advantage be used, in preference to the existing pattern, for general practical work. In the first place, as a result of its

low sensitivity to changes of shape, its construction calls for no great accuracy of workmanship ; small errors in the longitudinal disposition of the static holes, and quite large variations in the form of the nose, will not affect the calibration to any perceptible degree. It follows that all instruments of this type, made with ordinary workshop precision to the dimensions shown in Fig. 9, may be confidently accepted, without tests against a standard, as having the same calibration as that obtained in these experiments. That is to say, all such instruments will indicate a value of the velocity which is 0·3 per cent. too high, and if a head of this type is used in which the stem is entirely absent, an additional error of only 0·25 per cent. on velocity will be incurred. Both these errors are negligibly

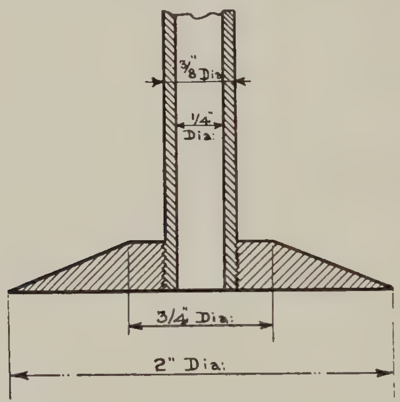



FIG. 12.—Disc type static head.

small for ordinary engineering practice, but in any case they are quite definite and due allowance may be made for them if desired. Further, this instrument is distinctly less sensitive than the existing standard pattern to errors of alignment with the wind direction, a feature of considerable importance in many practical cases where the direction of the current is not known to about 10° or so. Finally, the hemispherical head is more easily made and is much more robust than the present tapered type. The latter terminates in a thin feather edge which is very liable to become damaged, with the consequent possibility of a change in calibration.

**Disc Static Heads.**—In a dust-laden airstream the small holes in a static head of the tubular type are liable to become blocked, and for measurements under such conditions heads

of the type shown in Fig. 12 are sometimes used. Experiments by Heenan and Gilbert have shown * that these heads give all the accuracy ordinarily required in the measurement of static pressure. It is important that the face of the disc should be smooth and that the tube connecting the orifice to the mano-

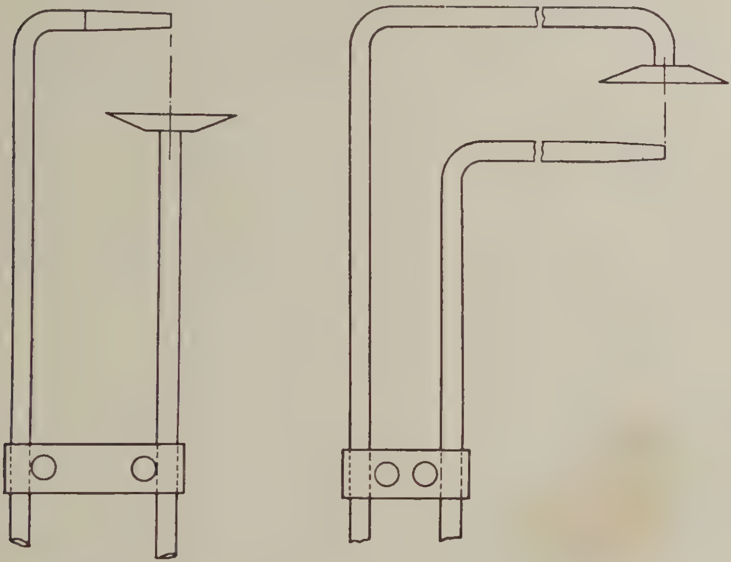


FIG. 13.—Combinations of disc static heads with total head tubes.

meter should end flush with the disc surface. Such heads are either used in conjunction with the usual form of total head tube, as illustrated in Fig. 13, where two possible arrangements are shown, or, alternatively, the measurements of static pressure and total head may be made separately as explained in Chapter V.

* *Proc. Inst. Civil Engineers*, vol. cxxiii., 1896, "The Design and Testing of Centrifugal Fans."

# CHAPTER IV.

## THE FLOW OF AIR IN PIPES.

Before considering practical methods of measuring the flow of air along a pipe, it will be useful, in order to define more closely the requirements which must be fulfilled, to discuss in some detail the nature of the flow. This subject appears to have been first studied from a general standpoint by Osborne Reynolds, whose classical researches * showed that two distinct types of flow are possible. Reynolds investigated the motion of water in straight horizontal glass tubes by means of a small quantity of colouring liquid introduced from a jet at a point on the axis near the inlet end. At low speeds of flow the colouring liquid was drawn out into a filament extending along the axis of the tube. As the speed was gradually increased the filament remained perfectly well defined until, when a certain speed was reached, it began to depart from linearity some distance from the mouth of the tube, and to show evidence of the formation of eddies in the water. When the speed was further increased the region of commencement of eddy formation approached nearer to the inlet of the tube, until finally the thread of colour completely lost its individuality and became diffused throughout the body of the water, in which turbulent motion had now become general. With a given size of tube and the water at a constant temperature, the speed at which eddy motion commenced was more or less constant ; but if the experiment was reversed, that is to say, the speed was gradually reduced from an initial value at which the motion was turbulent, the eddies were found to die out at a speed which was much more definite and somewhat lower than that at which eddy motion commenced for ascending speeds. The colour method could, of course, not be used for this reversed experiment ; the transition from turbulent to steady motion was detected in this case by observing the change

* *Phil. Trans. Roy. Soc.*, 1883.

in the relation between velocity and resistance * to flow in the pipe which occurs when the type of flow changes.

The speed at which steady flow changes to the turbulent variety is termed the higher critical velocity, and that at which the reverse effect takes place as the speed is reduced, the lower critical velocity. Since, as we have seen, the latter is the more definite, it is usually called simply the critical velocity, and this practice will be followed here.

We see, then, that in dealing with the flow of water in pipes (and it will be shown immediately that this applies also to the flow of air), we have two possible types of motion to consider. In the first the flow is of the stream-line character, the speed is slow and the inertia forces are small compared with the viscous or tangential frictional forces existing between neighbouring elements of the fluid. It appears, therefore, that for this type of motion viscosity is predominant in determining the nature of the flow, and it is, in fact, found that calculations of such motions, based purely on theoretical reasoning, agree accurately with experimental observations, the accord being so good that methods based on the slow stream-line flow of liquids through narrow tubes have been adopted for absolute determinations of viscosity. The other possible type of motion is turbulent; in this case the speed is higher and, in consequence, inertia forces can no longer be neglected.

It is necessary now to consider which type of flow occurs most commonly in engineering practice. For this purpose we shall require a definition of the coefficient of viscosity of a fluid. Let us take the case of a viscous fluid moving past a plane surface, the flow being everywhere parallel to the surface. The action of viscosity, or the tangential frictional forces between adjacent layers of the fluid, will be such as to tend to retard the faster moving layers and to accelerate those travelling more slowly. It follows that the fluid nearer the surface will move more slowly than that further away; it appears probable also that a very thin layer of fluid in immediate contact with the surface will be at rest, and actual observation confirms this deduction. A tangential force will therefore be exerted on the boundary, and a velocity gradient will exist in a direction normal to the surface. If $v$ is the velocity parallel to the surface at any distance $y$ measured perpendicular to the surface (Fig. 14), the velocity gradient at the surface (or the rate at

* For an explanation of the term resistance, see p. 35.

which the velocity increases as the distance from the surface increases) is given by $\frac{dv}{dy}$ * measured at the surface, and if F is the total tangential force acting on an area S of the surface, the coefficient of viscosity (usually denoted by $\mu$) of the fluid is defined as being such that

$$F = \mu S\left(\frac{dv}{dy}\right)_{\text{surface.}}$$

In general, we may, instead of considering the tangential force on the solid boundary, replace the latter by the surface of

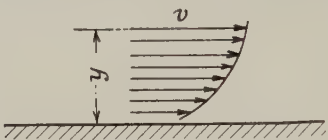



FIG. 14.

separation between any two contiguous layers of the fluid itself, and write

$$F = \mu S\frac{dv}{dy},$$

where $\frac{dv}{dy}$ is now the velocity gradient at the fluid surface in question.

From this we see that $\mu$ may be defined as " the tangential force on unit area of either of two horizontal planes of indefinite extent at unit distance apart, one of which is fixed, while the other moves horizontally with unit velocity, the space between being filled with the viscous substance." This definition is due to Maxwell.

We are now in a position to consider in greater detail the results of Reynolds' investigations. From experiments conducted in pipes of different diameters with water, for which the value of $\mu$ was altered by varying the temperature between 5° C. and 22° C., he deduced the fact, which is of fundamental importance, that the critical speed $v_c$ occurs at a constant value of the ratio $\frac{v_c d\rho}{\mu}$, where $d$ is the diameter of the pipe, and $\rho$ is

* If the velocity is proportional to the distance from the surface the gradient is, of course, simply $\frac{v}{y}$.

the density (mass per unit volume) of the water. He found also that this formula was applicable to other fluids besides water, and his and subsequent investigations have definitely established its general validity for the motion of all fluids along pipes.

Experiments to determine the magnitude of the constant value of $\frac{v_c d\rho}{\mu}$ at which the transition from turbulent to stream-line flow occurs agree in indicating an average figure of 2000 * for parallel pipes of ordinary smoothness, so that for any fluid

$$v_c \dagger = 2000 \frac{\mu}{d\rho}.$$

If we take the case of air at 15° C., $\frac{\mu}{\rho}$ is equal to about 0·00016 in foot-pound-second units (it should be remembered that $\rho$ is the weight per unit volume—in this case per cubic foot—divided by $g$, the gravitional constant) so that the critical velocity in feet per second is given by $\frac{0{\cdot}32}{d}$, where $d$ is in feet. If, for example, we consider a one-foot diameter pipe, the critical velocity for air at 15° C. occurs at 0·32 feet per second; alternatively, for air at 15° C. the critical velocity does not reach one foot per second until the pipe is about $3\frac{5}{8}$ inches in diameter.

It will be realised, therefore, that the stream-line flow of air in pipes is exceptional in practice, and that in the great majority of cases the flow will be turbulent. We shall therefore consider this type of flow more closely, and defer until the end of this chapter a short discussion of stream-line flow, which it has been thought advisable to include for the sake of completeness.

**The Resistance of a Pipe to the Motion of a Fluid along it.**—Let us consider first a mass $m$ of fluid moving along a horizontal pipe at a speed $v$, which is uniform across the section of the pipe, and subjected to a pressure (static pressure) $p$.

* The reader with a knowledge of dimensional theory will see that the quantity $\frac{v_c d\rho}{\mu}$ is non-dimensional, so that the value of the constant (2000) is applicable to any set of units.

† Throughout this chapter, unless otherwise stated, the velocity $v$ in a pipe is to be taken as the mean velocity, i.e. the volume of fluid passing in unit time divided by the cross-sectional area of the pipe.

Also let $\rho$, as before, denote the mass of unit volume of the fluid. The kinetic energy of the fluid is given by $\frac{1}{2}mv^2$ and the potential energy, i.e. the work necessary to bring the mass of fluid from a region of zero pressure to a region where the pressure is $p$, is $\frac{mp}{\rho}$, so that the total energy (referred to the mean level of the pipe) is given by

$$m\left(\frac{v^2}{2}+\frac{p}{\rho}\right),$$

i.e. by

$$V\left(\frac{\rho v^2}{2}+p\right),$$

since $m = \rho V$, where V is the volume of the mass $m$ of the fluid. Hence the energy per unit volume of the fluid is

$$\tfrac{1}{2}\rho v^2 + p.$$

It will be noted that each of the two terms in this expression has the dimensions of a pressure, and that their sum is, in fact, the total pressure of the fluid which would be measured by the facing tube of a Pitot and static tube. If there were no resistance to the motion of the fluid along the pipe, and no loss of energy due to the formation and dissipation of eddies, this total pressure would remain constant. Actually, of course, such dissipation of energy does occur, and there is also a resistance due to the viscous forces acting at the walls of the pipe, which causes a further loss of energy. If we consider two sections, one at each end of a straight length of pipe, we see that there will be a loss of energy as unit volume of the fluid passes from the upstream to the downstream section ; a corresponding amount of work will consequently have to be done in forcing this unit volume along the pipe. The resistance of the length of pipe between the two sections may therefore be taken as equal to this quantity of work. Further, since, as we have seen, the energy per unit volume passing a section is equal to the total head at that section, the resistance of the length of pipe will manifest itself by causing the total head at the downstream section to be less than that at the upstream section.

It is thus apparent that the resistance offered by a length of pipe to the passage of a volume of fluid along it may be measured in terms of the drop in total head experienced by the fluid in passing along this length, and in practice this method of expressing the resistance is universally adopted.

So far we have considered only flow in a parallel-walled straight pipe in which the velocities at all points in a cross-section are the same, so that the mean total head at that section can be obtained from a single measurement at any point in its plane. We shall see shortly, however, that such flow rarely, if ever, occurs in practice. Experiments show that the velocities are not in general the same at all radii in a given section of a parallel pipe along which a fluid is moving steadily.* It is found that the velocity is usually a maximum at the axis of the pipe and falls away towards the walls, and there seem to be definite indications that there is a very thin layer of the fluid in contact with the walls actually at rest. We must consider therefore how to measure resistance in an actual case, i.e. how to determine the mean total head across a section where the velocity is not uniform.

Before discussing the general case we may consider a straight parallel pipe, which presents no difficulties. In the first place, if the flow is parallel to the walls the static pressure across any cross-section must be constant. If this were not so the air would tend to flow from the high pressure regions to those of low pressure, and the resultant motion would not be parallel to the walls. Secondly, once the flow has settled down to the normal distribution in a straight pipe (see p. 42), that is, at a sufficient distance from the inlet, we may state that the distribution of velocity at all sections is the same.† Now the mean total head at any section will be equal to the static pressure $p$ at that section, which, being uniform, may be obtained by a single measurement anywhere in the section, together with the velocity head corresponding to the mean kinetic energy of unit volume of the air passing the section. If we denote the velocity corresponding to this mean kinetic energy by $\bar{v}$, the velocity head is $\frac{1}{2}\rho\bar{v}^2$.

Consider now a length of pipe whose end sections are denoted by A and B. The mean total head at A will be greater than the mean total head at B by a pressure $R_p$ which is equal to the work done on unit volume of the fluid in moving from A to B,

* Steadily is here used in the sense that the motion is unaffected by external influences such as the effect of the inlet to the pipe, or fluctuations in the speed of the fan or other actuator producing the motion. It is not to be taken as implying non-turbulent motion.

† Neglecting small effects such as the slight thickening of the retarded boundary layer of the fluid in contact with the walls, which occurs as the motion proceeds down the pipe.

i.e. to the resistance of the length AB of the pipe. We therefore have the equation,

$$\tfrac{1}{2}\rho\bar{v}_A^2 + p_A = \tfrac{1}{2}\rho\bar{v}_B^2 + p_B + R_p. \quad . \quad . \quad . \quad (1)$$

Now since, as already stated, the velocity distribution at all sections is the same for the assumed conditions, $\bar{v}_A = \bar{v}_B$, so that (1) becomes

$$R_p = p_A - p_B. \quad . \quad . \quad . \quad . \quad (2)$$

The physical significance of this equation is that, for steady flow in a length of straight pipe, the resistance is measured by the difference between the static pressures at the two ends of the length in question. It should be noted that $\rho$ at the two sections has been assumed to be the same, an assumption which is strictly true in the case of an incompressible fluid such as water, and sufficiently accurate for most purposes for such fluids as air, provided the pressure difference $p_A - p_B$ is small compared with either $p_A$ or $p_B$. Fortunately this condition is usually fulfilled in practical cases of air flow, so that equations (1) and (2) may be used as they stand.

Still confining ourselves to the case of the parallel pipe, we may consider the matter from another point of view. Suppose F is the frictional force resisting the motion, per unit area of the internal surface of the pipe, whose diameter is $d$. Then the total resistance of a length $l$ of the pipe is equal to

$$F\pi dl.$$

This frictional force tends to arrest the motion of the fluid along the pipe, and, for the flow to proceed steadily, there must be an equal force acting on the fluid in the pipe along the direction of motion. The latter force will be provided by the pressure difference between the two ends of the length $l$ of the pipe. Calling the pressures at the ends $p_A$ and $p_B$ as before, we see that the force on the volume of fluid contained at any instant by the length AB of the pipe (Fig. 15) will be given by

$$(p_A - p_B)\frac{\pi d^2}{4},$$

acting along the direction of motion. Hence

$$F\pi dl = (p_A - p_B)\frac{\pi d^2}{4},$$

or

$$p_A - p_B = \frac{4Fl}{d}. \quad . \quad . \quad . \quad . \quad (3)$$

The case of a pipe of varying cross-section is more complicated and, since the matter which follows immediately is con-

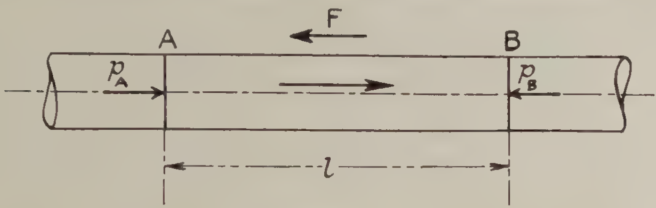



FIG. 15.

cerned with parallel pipes, consideration thereof has been deferred to the end of this chapter, in order not to interrupt the argument.

**The Characteristics of the Flow in Pipes.**—An extensive investigation of the flow of fluids along smooth straight pipes of drawn brass has been carried out by Dr. T. E. Stanton and the late Mr. J. R. Pannell,* the experiments being intended to establish values of F for different fluids flowing at various speeds in pipes of different diameters. The fluids used were air, water, and thick oil, whilst the pipes ranged from 0·12 inch to 5·0 inches in diameter and the speeds from 1 to 170 feet per second. The resistance F per unit surface area of each pipe for each condition of flow was determined by measuring the pressure drop over a given length of pipe, which was situated sufficiently far from the inlet to ensure that disturbances due to entry had died away before the working portion was reached. The results are of the greatest interest and practical importance. It was found that when the values of $\frac{F}{\rho v^2}$ were plotted against $\frac{vd}{\nu}$ †, all the observations, irrespective of the fluid used, the diameter of the pipe, or the mean speed of flow, fell on a single curve; in no case was the departure from the mean curve greater than could be accounted for by the limit of the experimental accuracy. The curve consisted of two parts (see Fig. 16), of which one—the left-hand portion—corresponded to values of $\frac{vd}{\nu}$ below about 2000, i.e. where the flow was of a

* *Phil. Trans. Roy. Soc.*, A., 1914, vol. 214, p. 199.

† The symbol $\nu$ is extensively used to represent the ratio $\frac{\mu}{\rho}$, which is called the "kinematic viscosity" of the fluid.

stream-line character, and the other to values of $\frac{vd}{\nu}$ above this, for which the flow was turbulent. The two branches of the curve were connected by a region of instability, corresponding roughly to values of $\frac{vd}{\nu}$ of 2000 to 3000, which marked the transition from one type of flow to another.

**Turbulent Flow in Pipes.**—As we have already seen, turbulent flow is the type with which engineers are almost invariably concerned, so that the portion of the curve for values of $\frac{vd}{\nu}$ in excess of about 3000 is that which is of immediate interest. Dr. C. H. Lees,* from an examination of the data given by Stanton and Pannell, together with those of other investigators, has proposed the following formula for the resistance per unit area :—

$$F = \rho v^2\left\{0{\cdot}0009 + 0{\cdot}0763\left(\frac{\nu}{(vd)}\right)^{0{\cdot}35}\right\}, \qquad . \quad (4)$$

whence, by the use of equation (3), we obtain

$$p_A - p_B = \frac{\rho v^2 l}{d}\left\{0{\cdot}0036 + 0{\cdot}305\left(\frac{\nu}{vd}\right)^{0{\cdot}35}\right\}, \qquad . \quad (5)$$

where $p_A$ and $p_B$ are respectively the pressures at the ends A and B of the length $l$ of the pipe.

The curve deduced from equation (4) coincides accurately with the mean curve through the points obtained by Stanton and Pannell, and the portion of Fig. 16 pertaining to turbulent flow has been calculated in this way.

Equation (5) is of practical importance since it enables the resistance of smooth parallel pipes to be calculated for any condition of flow. It should be noted that the resistance, or pressure drop per unit length, does not vary as the square of the speed, since, if this were the case, the quantity

$$\left\{0{\cdot}0036 + 0{\cdot}305\left(\frac{\nu}{vd}\right)^{0{\cdot}35}\right\}$$

would then have a constant value, and the curve in Fig. 16 would be a straight line parallel to the axis of $\frac{vd}{\nu}$. It will be noted, however, that the curve, although not parallel to this

* *Proc. Roy. Soc.*, A., 1915, vol. 91, p. 46.

axis, tends to become so as the value of $\frac{vd}{\nu}$ increases, so that the error involved by assuming that the frictional resistance varies as the square of the speed becomes less at the higher values of $\frac{vd}{\nu}$. The highest value of $\frac{vd}{\nu}$ shown on the curve is about 400,000, which corresponds to a velocity of about 65 feet per second for air at 15° C. in a one-foot diameter pipe. In practice

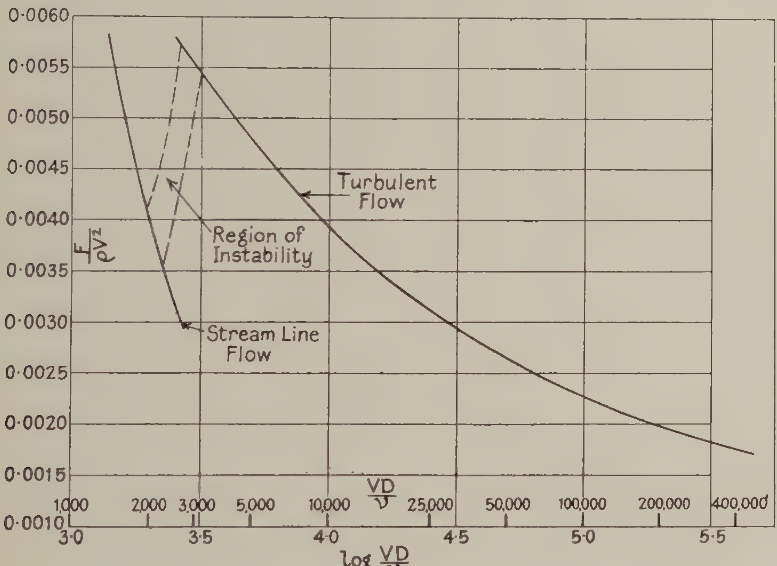



FIG. 16.—Resistance of smooth circular pipes.

it is often convenient to assume that F varies as some power $n$ of $v$, and Lees * has deduced the following empirical expression for the index $n$, which fits Stanton and Pannell's results with good accuracy :—

$$n = 2 - \frac{29\cdot 7}{\left[85 + \left(\frac{vd}{\nu}\right)^{0\cdot 35}\right]} \quad . \quad . \quad . \quad . \quad (6)$$

For smooth parallel pipes we may therefore assume that $F = kv^n$ and calculate $n$ by the use of (6). For $\frac{vd}{\nu} = 3000$,

* *Loc. cit.*

where the region of turbulent flow may be regarded for practical purposes as commencing, $n$ will be found to be 1·71, and for $\frac{vd}{\nu} = 400{,}000$ it becomes 1·83. These values are strictly applicable only to smooth-walled pipes: the question of rough pipes is more complicated, as the degree of roughness affects the results to a material extent. It is found that as the roughness increases $n$ approaches the value 2, in which case $\frac{F}{\rho v^2}$ will have a constant value, which appears to be somewhat higher than the corresponding value for smooth pipes. For the pipes ordinarily used in practice, provided they are reasonably smooth, the curve of Fig. 16 and the values of $n$ obtained from equation (6) will usually be found to provide a sufficiently good basis for estimating the frictional resistance.

**Velocity Distribution Across a Section.**—The form of the curve of velocity distribution for a smooth pipe as obtained by Stanton is shown in Fig. 17. Stanton found that up to a radius equal to 0·8R, where R is the internal radius of the pipe, the distribution curve for a smooth pipe is parabolic, so that the velocity $v_r$ at a radius $r$ (where $r$ is less than 0·8R) is given by

$$v_r = v_a - Ar^2,$$

where $v_a$ is the velocity at the axis and A is a constant.*

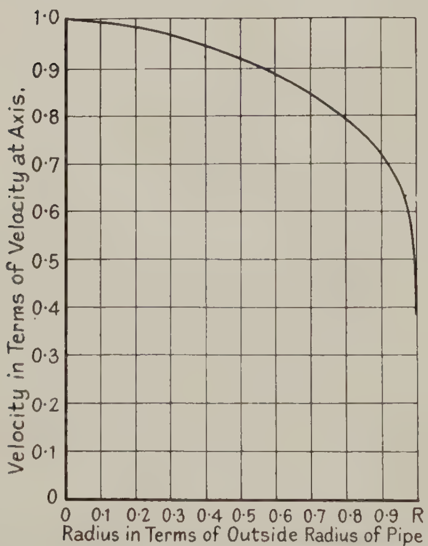



FIG. 17.—Distribution of velocity across smooth circular pipes.

In Stanton and Pannell's research on wall friction, to which reference has been made above, it was, in many instances, found convenient to deduce the mean velocity from a single reading of the velocity $v_a$ at the axis measured by means of a Pitot tube, the static pressure being obtained from a hole in the side

* *Proc. Roy. Soc.*, 1911, A., vol. 85, p. 366.

of the pipe.* For this purpose it was necessary to determine accurately the relation between $v$ † and $v_a$ for all conditions of flow, and a comprehensive investigation of this relationship was undertaken as a preliminary to the major research on wall friction. Although the results obtained were thus subsidiary to the main experiments, they furnish data which is at least of equal value to the engineer, since they enable him to obtain the mean speed of flow in a long parallel pipe from a single reading of velocity taken at the axis.* Fig. 18 exhibits the results in graphical form. When the values of $\frac{v}{v_a}$ were plotted against $\frac{vd}{\nu}$ it was found that all the points fell on the mean curve reproduced in Fig. 18, irrespective of the fluid

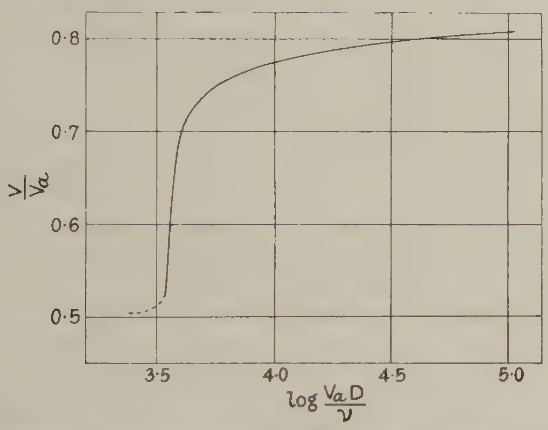



FIG. 18.—Relation between mean and axial velocity in smooth circular pipes.

(air or water) or the diameter of the pipe. For values of $\frac{vd}{\nu}$ below the critical, the ratio was found to be 0·5, which, as we shall see shortly, is the theoretical value for viscous streamline flow. At the highest value of $\frac{vd}{\nu}$ obtainable the value of the ratio was 0·81. This curve can be employed with confidence to give a good approximation to the mean speed from a measurement of $v_a$, provided that the section at which the

* See Chapter VIII. † See footnote † to p. 35.

measurement is made is, as was actually the case in the experiments under consideration, at a considerable distance from the inlet to the pipe, so that disturbances originating there may have had time to die out. Stanton and Pannell took their observations at distances varying from 90 to 140 diameters from the inlet, since previous experiments had shown that disturbances may persist for a considerable distance along the pipe. In practice the section should be at least some 40 or 50 diameters from the inlet. Although this restriction constitutes a serious objection which may often preclude the use of this method for determining the mean velocity of flow, cases will frequently occur in practice where the necessary conditions are fulfilled, and the method may be adopted.

In the case of rough pipes, where the frictional resistance varies as the square of the speed, Stanton found * that the velocity distribution was parabolic up to a radius very nearly equal to the inner radius of the pipe, and that this distribution did not, as it did in the case of the smooth pipes, vary with the value of $\frac{vd}{\nu}$. The ratio $\frac{v}{v_a}$ for these rough pipes had the value of about 0·76. It should be noted that the pipes in question were artificially roughened, by cutting double screw threads on the internal surfaces, in order that the frictional resistance should vary as the square of the speed—a point that was verified experimentally. It will be realised, therefore, that the pipes were appreciably rougher than those commonly employed in practice, and it would thus appear that, subject to the restriction previously specified, the curve of Fig. 18 relates sufficiently closely to ordinary conditions to justify its practical application.

**Stream-Line Flow in Pipes.**—Let us imagine a fluid flowing along a straight parallel pipe under the influence of viscous forces only. If we consider any cylindrical element AB (Fig. 19) of the fluid, coaxial with the pipe, we see that there will be a viscous drag, which opposes the motion, acting on the cylindrical surface, and for the motion to proceed steadily this drag must be balanced by the difference in the pressures acting on the two ends A and B.

Let the length of the cylindrical element be $dl$ and its radius $r$, and let R be the radius of the pipe.

Since the motion is assumed to be parallel to the walls of

* *Proc. Roy. Soc.*, 1911, A., vol. 85, p. 366.

the pipe, there are no viscous forces acting radially and therefore the pressure across any section perpendicular to the axis is uniform. We may therefore denote by $p$ the pressure on the face A, and that on B by $p - dp$. The pressure difference

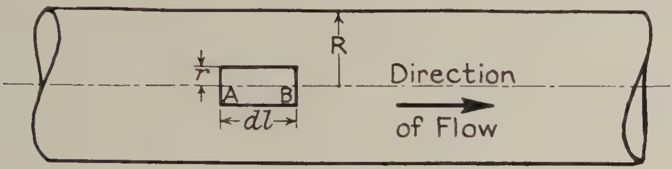



FIG. 19.

maintaining the motion is therefore $\pi r^2 dp$, and this must be equal to the viscous retarding force acting on the curved surface of the cylinder, which, from the definition of the coefficient of viscosity $\mu$, is $2\pi r dl \mu \frac{dv}{dr}$, where $v$ is the velocity at radius $r$.

Hence

$$\pi r^2 dp = - 2\pi r dl \mu \frac{dv}{dr},$$

or

$$\frac{dp}{dl} = - \frac{2\mu}{r} \frac{dv}{dr}.$$

Now the velocity will be some function of the radius only, since $\mu$ is constant and the flow is continuous and parallel to the walls of the pipe. In the above equation, therefore, the right-hand side is a function of $r$ only, and since the pressure does not vary across the section it follows that $\frac{dp}{dl}$ is a constant for given conditions of flow. We may therefore write $\frac{p_1 - p_2}{l}$ for $\frac{dp}{dl}$, where $p_1$ and $p_2$ are now the pressures at two sections at a finite distance $l$ apart ; i.e.

$$\frac{p_1 - p_2}{l} . r dr = - 2\mu dv,$$

and, on integration between $r = 0$ and $r = R$, we have

$$\frac{p_1 - p_2}{l} . \frac{R^2}{2} = - 2\mu(v_R - v_a), \qquad (7)$$

where $v_a$ is the velocity at the axis.

If we assume that the velocity is zero at the walls,

$$v_R = 0,$$

and

$$\frac{p_1 - p_2}{l} = \frac{4\mu v_a}{R^2}. \qquad (8)$$

The velocity at any radius $r$ is obtained from (7) by substituting $r$ for R and inserting the value of $v_a$ from (8), and will be seen to be given by

$$v_r = \frac{p_1 - p_2}{4\mu l}(R^2 - r^2), \qquad (9)$$

from which it appears that the velocity distribution is parabolic across the section.

The mean velocity is obtained as follows :—

Imagine an annulus of the pipe at a radius $r$ and of thickness $dr$; the volume of fluid flowing through the annulus is given by $2\pi r dr v_r$, where $v_r$ is obtained from (9). The total volume flowing is therefore

$$V = \int_0^R 2\pi r v_r dr = \frac{\pi(p_1 - p_2)}{2\mu l}\int_0^R r(R^2 - r^2)dr$$

$$= \frac{\pi(p_1 - p_2)}{2\mu l}\left[\frac{r^2R^2}{2} - \frac{r^4}{4}\right]_0^R = \frac{\pi(p_1 - p_2)R^4}{8\mu l}.$$

Now the mass of fluid passing in unit time is obviously constant, and if the pressure drop is small compared with either $p_1$ or $p_2$ (which are usually, of course, approximately atmospheric) we may, in the case of a gas, neglect changes of density and regard the volume flowing in unit time as constant. In the case of liquids, which are practically incompressible, this will be so even if the pressure drop is large. In either case, therefore, the volume flowing in unit time may be taken as constant and equal to $\pi R^2 v_m$, where $v_m$ is the mean velocity of flow.

Hence

$$\pi R^2 v_m = \frac{\pi R^4(p_1 - p_2)}{8\mu l},$$

or

$$v_m = \frac{p_1 - p_2}{8\mu l}R^2. \qquad (10)$$

Now (8) may be written

$$v_a = \frac{p_1 - p_2}{4\mu l}R^2.$$

In stream-line flow, therefore, the mean velocity of flow is half the velocity at the axis.

**The Resistance of a Pipe of Varying Cross-section.**—It has been shown that the resistance of any given length of pipe is equal to the loss in energy experienced by unit volume of the fluid in passing along this length. When the pipe is parallel between the two sections considered and the velocity distributions at these sections are the same, we have seen that the resistance is equal to the difference of static pressure between the sections; but if the diameters of the two ends of the length of pipe are not the same the resistance is equal to the difference between the mean energy of unit volume of the fluid at the two sections. The same is true for a parallel pipe in which the velocity distributions at the two ends are not identical. In the general case we have therefore to measure the mean energy per unit volume, i.e. the mean total head, at the two sections and to subtract one from the other. This may be done as follows:—

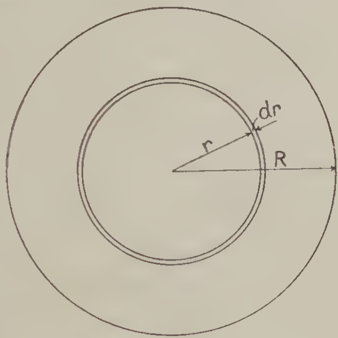


FIG. 20

Consider any section of the pipe of radius R, and let $v$ be the mean velocity at radius $r$ through an annular element of width $dr$. Also let $p$ be the static pressure at this radius ($p$ is taken above or below atmospheric pressure in the usual manner). The volume of fluid passing through the annulus in unit time is $2\pi r dr v$; its mass is therefore $2\pi r dr . \rho v$, and its kinetic energy $2\pi r dr . \rho v . \frac{v^2}{2}$. The potential energy is $2\pi r dr v p$. Hence the total energy is equal to $\pi r \rho v^3 dr + 2\pi r v p dr$. The total energy of the fluid flowing across the section per second is therefore

$$\pi\rho\int_0^R v^3 r dr + 2\pi\int_0^R p v r dr.$$

Also the total volume flowing per second is $2\pi\int_0^R vrdr$, so that the energy per unit volume, or the mean total head, is equal to

$$\frac{1}{2}\rho\frac{\int_0^R v^3rdr}{\int_0^R vrdr}+\frac{\int_0^R pvrdr}{\int_0^R vrdr}.$$

In order, therefore, to obtain the mean total head at a section the values of the above integrals must be determined by taking sufficient measurements of $v$ and $p$ across the section. Thus, in order to determine, for example, $\int_0^R pvrdr$, $p$ and $v$ must be measured at a number of values of $r$ ; a curve of the product $pvr$ is then plotted on an $r$ base, and its area gives the value of the integral. The units for the various quantities must be consistent : thus if $v$ is in feet per second, $p$ must be in pounds per square foot(= $5{\cdot}2\times$ inches of water), $r$ in feet, and $\rho$ in $\dfrac{\text{pounds per cubic foot}}{32{\cdot}2}$. The resulting total head will be in pounds per square foot. The total heads at each of the two sections forming the ends of the length of piping whose resistance is required must be obtained in this way, and their difference will be the resistance.

Usually it will be found that, although the pipe diameters at the two sections are different, the sections themselves nevertheless form parts of parallel-walled pipes, so that the flow is axial and $p$ may be taken as constant across the section. The integral $\int_0^R pvrdr$ is then equivalent to $p\int_0^R vrdr$, so that the expression for the mean total head at a section reduces to

$$\frac{1}{2}\rho\,\frac{\int_0^R v^3rdr}{\int_0^R vrdr}+p. \qquad . \qquad . \qquad . \quad (11)$$

It may be noted that if $v$ is constant across the section it may be taken outside the integrals in which it occurs and the expression reduces to its familiar form of $\frac{1}{2}\rho v^2+p$.

Now in fan testing and similar work, resistance is a quantity which has frequently to be determined, in addition to the volume of air flowing per second. The latter is deduced by measuring the mean velocity $v_m$ across the section by one of the methods described in subsequent chapters. It is then common practice to say that the mean total head at the section, for the purpose of computing resistance, is equal to $\frac{1}{2}\rho v_m{}^2 + p$. It will be apparent that this statement is erroneous, and in view of its wide acceptance it is important to examine the magnitude of the possible errors involved.

As a concrete example we may take the case of the smooth parallel pipe for which Stanton measured the velocity distribution as given in Fig. 17. The following table shows the values from which this curve was obtained; here $v$ is the velocity at radius $r$, $v_a$ is the velocity at the axis and R is the internal radius of the pipe The integrals occurring in (11) can now

TABLE I.

| $\frac{r}{R}$. | $\frac{v}{v_a}$. | $\frac{r}{R}$. | $\frac{v}{v_a}$. |
|---|---|---|---|
| 0 | 1·000 | 0·799 | 0·789 |
| 0·183 | 0·987 | 0·852 | 0·753 |
| 0·284 | 0·967 | 0·905 | 0·711 |
| 0·389 | 0·944 | 0·925 | 0·688 |
| 0·491 | 0·915 | 0·954 | 0·645 |
| 0·596 | 0·883 | 0·978 | 0·596 |
| 0·698 | 0·838 | 0·990 | 0·388 |

be evaluated graphically thus:—

For $\int_0^R v^3 r dr$ we may substitute its equivalent

$$v_a{}^3 R^2 \int_0^R \left(\frac{v}{v_a}\right)^3 \frac{r}{R} \frac{dr}{R};$$

similarly $\int_0^R v r dr$ may be replaced by

$$v_a R^2 \int_0^R \frac{v}{v_a} \frac{r}{R} \frac{dr}{R}.$$

If we now plot on a base of $\frac{r}{R}$ the values of $\left(\frac{v}{v_a}\right)^3\frac{r}{R}$ and $\frac{v}{v_a} \cdot \frac{r}{R}$ obtained from the above table, we shall derive two curves whose areas are respectively the two integrals required. Thus we shall find that

$$v_a{}^3R^2\int_0^R\left(\frac{v}{v_a}\right)^3\frac{r}{R}\frac{dr}{R} = 0{\cdot}291v_a{}^3R^2, \quad . \quad (12a)$$

and

$$v_aR^2\int_0^R\frac{v}{v_a}\frac{r}{R}\frac{dr}{R} = 0{\cdot}405v_aR^2, \quad . \quad . \quad (12b)$$

so that the true mean total head, from (11), becomes

$$\tfrac{1}{2}\rho \times 0{\cdot}718v_a{}^2 + p.$$

Let us compare this with the mean total head erroneously, but frequently, stated to be $\frac{1}{2}\rho v_m{}^2 + p$. In this case

$$v_m = \frac{2\pi\int_0^R vrdr}{\pi R^2} = \frac{2v_aR^2\int_0^R \frac{v}{v_a}\frac{r}{R}\frac{dr}{R}}{R^2}$$

$$= 0{\cdot}81v_a \text{ (from (12}b\text{))}.$$

Hence $$v_m{}^2 = 0{\cdot}656v_a{}^2,$$

and the mean total head calculated in this way is

$$\tfrac{1}{2}\rho \times 0{\cdot}656v_a{}^2 + p.$$

The true mean total head therefore exceeds that calculated by the usual method by $0{\cdot}062 \times \frac{1}{2}\rho v_a{}^2$, i.e. by $0{\cdot}095 \times \frac{1}{2}\rho v_m{}^2$. Now the head corresponding to the mean velocity $v_m$ is, of course, $\frac{1}{2}\rho v_m{}^2$, so that we may say that the error incurred in estimating the mean total head by this incorrect method may be as much as $9\frac{1}{2}$ per cent. of the head corresponding to the mean velocity. The smaller the velocity at a section in relation to the static pressure, the less will be the error on total head. If we take the case, which is of quite common practical occurrence, in which the velocity head (mean velocity) is about equal to the mean static pressure at a section, we see that the mean total head deduced by the usual method may be about 5 per cent. too low—an error of by no means negligible magnitude.

Methods of measuring the true mean total head at any section in practice are described in Chapter V.

# CHAPTER V.

## MEASUREMENTS OF FLOW AND RESISTANCE WITH PITOT-STATIC TUBES.

It was shown in the last chapter that the speed of flow at all points in a cross-section of a pipe is not in general the same, the velocity being a maximum at the centre and falling away towards the walls. Now instruments such as the combined Pitot-static tube or the vane anemometer will, of course, only indicate the local velocity in the regions which they occupy, so that in order to determine the volume of air flowing in unit time by the use of such instruments, it is necessary to measure the velocity at a number of points across a given section. Before we deal with the use of any particular instrument it will be well to discuss the problem from a general standpoint.

**Determination of the Mean Velocity.**—We shall, as before, define the mean velocity of flow $v_m$ in the pipe as that velocity which, when multiplied by the area A of the pipe section, gives the volume V of air passing through the section per second.

If we consider a small element of area $a_1$ of the pipe through which the mean velocity of flow is $v_1$, the volume flowing through this element is $a_1v_1$, and

$$V = \Sigma a_1 v_1 = A v_m, \quad . \quad . \quad . \quad (1)$$

so that $$v_m = \frac{1}{A} \Sigma a_1 v_1.$$

Suppose now that we divide the section of the pipe into $n$ equal parts of area $a$, through which the mean velocities are $v_1$, $v_2$, $v_3$, etc., then we shall have

$$v_m = \frac{1}{A} \Sigma a v_1 = \frac{1}{A}(a v_1 + a v_2 + a v_3 + \ldots + a v_n)$$
$$= \frac{a}{A}(v_1 + v_2 + \ldots + v_n),$$

so that, since $A = na$,

$$v_m = \frac{1}{n}(v_1 + v_2 + \ldots + v_n). \qquad (2)$$

We see therefore that the mean velocity of flow through the pipe is obtained by dividing the section into a number of parts of equal area and measuring the mean velocity through each. If the number of parts is sufficiently large, i.e. if each part is sufficiently small, the mean velocity through any part may be taken as the velocity of flow at its centre. The number of velocity readings necessary will therefore be equal to the number of parts into which the section is divided. It will be clear that the more irregular the velocity distribution across the section, the greater will be the number of parts into which the section must be divided, if the velocity measured at the centre of area of each part is to approximate to the mean velocity across that part. Further, if we consider two pipes, similar in section but different in size, and assume that the velocity distributions across any pair of corresponding sections are similar, then for the same percentage accuracy in the determination of $v_m$, the two sections must evidently be divided into the same number of corresponding parts. This point is often not appreciated; it must be remembered that if, for practical reasons, fewer measurements are taken in a small pipe, some accuracy will be sacrificed. The number of readings taken should depend primarily on the nature of the velocity distribution across the pipe, and not on the size of the pipe. At the same time, more readings should be taken in large pipes, unless the velocity distribution takes the form of a smooth curve, for the reason that if the points of measurement are too far apart, any local irregularities in velocity are liable to be passed over in the observations, and such irregularities are perhaps likely to be more important in large pipes than in small ones.

In considering how best to divide up a pipe section in practice, we need discuss only pipes of circular and rectangular cross-section, since it is exceptional to use sections of any other form.

(*a*) **Pipes of Rectangular Cross-section.**—The section should be divided, by lines parallel to the sides, into a number of equal areas each geometrically similar to the whole section (Fig. 21), and the velocity should be measured at the centre of each. The mean velocity and the volume flowing per minute can then be obtained directly by the use of equations (1) and

(2). As regards the number of parts into which the section should be divided, a good working rule, subject to the remarks made above, is to make the minimum number 16 for pipes up to 576 square inches in cross-sectional area, and for larger pipes to take the number such that each part is not greater than 36 square inches in area. (For pipes of less than 5-inch side see remarks for small circular pipes.)

(*b*) **Pipes of Circular Cross-section.**—The pipe should be divided into $n$ equal concentric zones with centres at O, as in Fig. 22, by circles of radii $r_2$, $r_4$, $r_6$, etc. If now we take

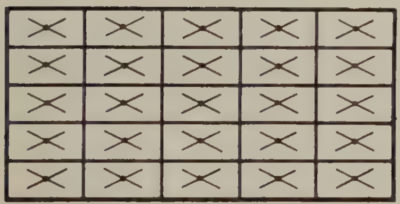



FIG. 21.

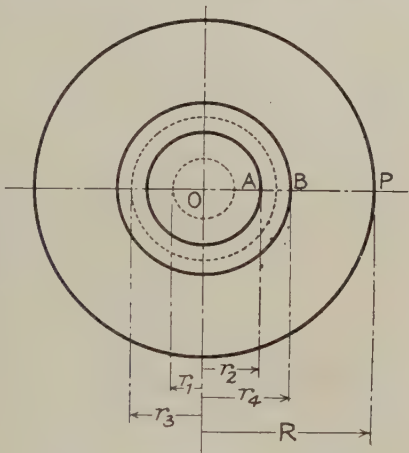



FIG. 22.

any radius OP of the pipe, and consider a particular zone—AB say—a velocity measurement must be made along this radius at a point such that the circle through it with O as centre divides AB into two equal areas. The positions of the points of measurement in the other zones are determined in the same manner, and it will be evident that the whole section is in effect divided into $2n$ parts by circles of radii $r_1$, $r_2$, $r_3$, etc., and that $n$ velocity measurements are made at the radii $r_1$, $r_3$, $r_5$ and so on, the outermost point of observation being $r_{2n-1}$.

If R is the length of the radius of the pipe, the following equations express the conditions that all the zones are of equal area :—

$$\pi(R^2 - r^2_{2n-1}) = \pi(r^2_{2n-1} - r^2_{2n-2}) = \text{etc.}$$
$$= \pi(r_2{}^2 - r_1{}^2) = \pi r_1{}^2,$$

so that

$$r_2^2 = 2r_1^2,$$

$$r_3^2 = 3r_1^2, \text{ etc.},$$

$$r^2_{2n-1} = (2n - 1)r_1^2,$$

and

$$R^2 = 2nr_1^2.$$

Hence

$$r_1 = R\sqrt{\frac{1}{2n}},$$

$$r_2 = R\sqrt{\frac{2}{2n}}, \text{ etc.},$$

and

$$r_{2n-1} = R\sqrt{\frac{2n-1}{2n}}.$$

The $n$ readings of velocity are therefore to be taken at radii $R\sqrt{\frac{1}{2n}}$, $R\sqrt{\frac{3}{2n}}$, $R\sqrt{\frac{5}{2n}}$, etc., the final radius being $R\sqrt{\frac{2n-1}{2n}}$.

In practice it is not sufficient to take readings along one radius only. The mean velocity for each zone should be obtained from four readings, one along each of four radii of which two form one pipe diameter and the other two the perpendicular diameter. It follows that $2n$ readings of velocity will be taken along each of the two chosen diameters, making a total number of $4n$ observations, and it will usually be found convenient to locate the positions of observation by measuring from one end of the diameter. Expressed in this way, the distances at which readings of velocity are to be taken will be

$$\frac{D}{2}\left(1 - \sqrt{\frac{2n-1}{2n}}\right), \frac{D}{2}\left(1 - \sqrt{\frac{2n-3}{2n}}\right), \text{ etc.,}$$

$$\text{up to } \frac{D}{2}\left(1 - \sqrt{\frac{1}{2n}}\right)$$

on one side of the centre of the pipe, and

$$\frac{D}{2}\left(1 + \sqrt{\frac{2n-1}{2n}}\right), \frac{D}{2}\left(1 + \sqrt{\frac{2n-3}{2n}}\right), \text{ etc.,}$$

$$\text{up to } \frac{D}{2}\left(1 + \sqrt{\frac{1}{2n}}\right)$$

on the other, where D is the internal pipe diameter.

The number of observations to be taken will be governed largely by the considerations set out above. The following practical rules will, however, be found to serve as a useful guide in most cases.

For pipes of 6 inches to 12 inches diameter, $n$ should have the value 3, so that six readings of velocity will be taken along each of two mutually perpendicular diameters, at distances of 0·044D, 0·147D, 0·296D, 0·704D, 0·854D, and 0·957D from the inside wall of the pipe.

For pipes over 12 inches diameter, $n$ should be not less than 5, when ten observations will be made along each diameter at distances from the wall of 0·026D, 0·082D, 0·147D, 0·226D, 0·342D, 0·658D, 0·774D, 0·854D, 0·919D, and 0·975D. In cases where the diameter of the pipe exceeds 3 feet, more than 5 points along a radius should be taken.

Special consideration is needed for the accurate measurement of the flow in small pipes of less than 6 inches diameter; this subject is discussed more fully in Chapter VIII.

The above rules for determining the number of areas into which pipes should be divided must not be regarded as rigidly applicable in all cases. They will be found to give reasonably accurate results provided that the velocity distribution across a diameter of the pipe follows a more or less smooth curve, but if any doubt is felt as to the nature of the distribution in a particular case, a preliminary exploration, comprising velocity measurements at a number of points in the section, should always be made. If the velocity distribution is found to be unduly irregular, and it is not possible to make measurements at some other section of the pipe where conditions are more favourable, there will be no alternative but to take a larger number of readings than the above rules indicate.

**Measurements with Pitot-Static Tubes.** — Measurements of velocity head must be made either by means of the combined type of Pitot-static tube or by means of separate total and static tubes inserted successively at the appropriate points as described below. In either case the resultant reading at each point of measurement will be a pressure $p$ which is related, as we have seen, to the speed $v$ at the point of measurement by the equation,

$$p = \tfrac{1}{2}\rho v^2, \qquad \ldots \qquad (3)$$

where, if $v$ is to be in feet per second, $\rho$ must be expressed in

slugs per cubic foot $\left(\frac{\text{pounds weight per cubic foot}}{g}\right)$ and $p$ in pounds per square foot.

For air at a temperature of 15·6° C. (60° F.) and a pressure of 760 mm. of mercury (29·92 inches) $\rho = 0{\cdot}00237$, and for any other temperature $t$° C. and pressure $b$ mm. of mercury,

$$\rho = 0{\cdot}00237 \times \frac{288{\cdot}6}{273 + t} \times \frac{b}{760}$$

$$= 0{\cdot}009 \times \frac{b}{273 + t}. \qquad (4)$$

Usually, it is more convenient to measure $p$ not in pounds per square foot but as $h$ inches of water column, so that

$$h = p \times \frac{12}{62{\cdot}36} = 0{\cdot}1924p.$$

Hence if the velocity head $h$ is expressed in inches of water (3) becomes

$$h = 0{\cdot}0000866v^2 \times \frac{b}{273 + t},$$

or

$$v = k\sqrt{h}, \qquad (5)$$

where

$$k = 107{\cdot}5\sqrt{\frac{273 + t}{b}}.$$

If $t$ is measured in degrees Fahrenheit and $b$ in inches of mercury, the corresponding value of $k$ becomes

$$K = 15{\cdot}88\sqrt{\frac{460 + t}{b}}.$$

Returning now to equation (2), we see that, when Pitot and static tubes are used to measure the quantity of air flowing in a pipe, the average velocity $v_m$ is given by

$$v_m = \frac{k}{n}(\sqrt{h_1} + \sqrt{h_2} + \sqrt{h_3} + \ldots), \qquad (6)$$

where $h_1$, $h_2$, $h_3$, etc. are the velocity heads observed at the points of measurement, whose positions are determined in the manner explained above. If we denote by $h_m$ the velocity head corresponding to the average velocity we have, from (6),

$$\sqrt{h_m} = \frac{v_m}{k} = \frac{1}{n}(\sqrt{h_1} + \sqrt{h_2} + \sqrt{h_3} + \ldots); \qquad (7)$$

i.e. the square root of the head equivalent to the mean velocity is equal to the arithmetic mean of the square roots of the velocity heads observed at the centres of zones of equal area.

The volume of air passing per second in cubic feet will be

$$\frac{Ak}{n}(\sqrt{h_1}+\sqrt{h_2}+\sqrt{h_3}+\ldots), \text{ or } Ak\sqrt{h_m},$$

and the weight of air, in pounds per second,

$$\frac{Ag\rho k}{n}(\sqrt{h_1}+\sqrt{h_2}+\sqrt{h_3}+\ldots), \text{ or } Ag\rho k\sqrt{h_m},$$

where $\rho$ is obtained for the appropriate conditions of temperature and pressure from equation (4), and A is the area of the pipe in square feet.

In practice, engineers usually prefer to measure the flow in cubic feet V or pounds Q per minute. For these units we shall find

$$\left.\begin{aligned} V &= A \times 6450\sqrt{\frac{273+t}{b}}\sqrt{h_m} \text{ cubic feet per minute,} \\ \text{and } Q &= A \times 186{\cdot}9\sqrt{\frac{b}{273+t}}\sqrt{h_m} \text{ pounds per minute,} \end{aligned}\right\} \quad (8)$$

when temperature and pressure are measured in degrees Centigrade and millimetres of mercury respectively.

When the Fahrenheit and inches of mercury scales are used, these expressions become

$$\left.\begin{aligned} V &= A \times 953\sqrt{\frac{460+t}{b}}\sqrt{h_m} \text{ cubic feet per minute,} \\ \text{and } Q &= A \times 1264\sqrt{\frac{b}{460+t}}\sqrt{h_m} \text{ pounds per minute,} \end{aligned}\right\} \quad (9)$$

It will be seen that the expressions for both V and Q involve a term containing $\sqrt{b}$, in one case in the denominator and in the other in the numerator, so that a 2 per cent. error in $b$ will give rise to a 1 per cent. error in V or Q. Now there will, in general, be a static pressure $p_s$ in the pipe, and according to the usual convention, which we shall adopt here and in subsequent chapters, this pressure is measured above or below the atmospheric pressure $p_a$*. The absolute static pressure of the

* According to this convention when the static pressure is said to be zero, the air is actually at atmospheric pressure.

air will thus be equal to $p_s + p_a$, and this is the quantity we have denoted by $b$ in equations (8) and (9). The average barometric pressure $p_a$ may be taken as roughly equal to 400 inches of water, so that unless the measured static pressure $p_s$ in a particular case numerically exceeds 8 inches of water, we may neglect it in comparison with the atmospheric pressure $p_a$ without causing an error exceeding 1 per cent. in V or Q. It follows that in a large number of cases $b$ in the above formulæ can be taken as the barometric pressure $p_a$ without sensible error. If, however, greater accuracy is required than is afforded by this assumption, $b$ must be taken as equal to $p_a$ plus or minus the static pressure $p_s$, according as the latter is positive or negative. In making this correction it should be remembered that $p_s$ is usually measured in inches of water whereas $b$ is to be expressed in millimetres or inches of mercury according as formula (8) or (9) is being used. For (8) $p_a$ must be increased or diminished by $\frac{p_s}{13{\cdot}56 \times 25{\cdot}4}$, i.e. by $0{\cdot}0029p_s$, and for (9) by $\frac{p_s}{13{\cdot}56}$ or $0{\cdot}074p_s$.

**Summary of Procedure for Measuring Quantity with Pitot and Static Tubes.**—From the foregoing we may summarise as follows the procedure to be followed in obtaining by means of Pitot and static tubes the quantity of air flowing in a pipe of cross-sectional area A square feet :—

(1) Measure the velocity head in inches of water at a number of points across the section, determined in the manner already indicated, which are at the centres of zones of equal area.

(2) Determine the square root of each velocity head and divide the sum of the square roots by the number of observations. In this manner the square root of the head equivalent to the mean velocity is obtained.

(3) Denoting the head equivalent to the mean velocity by $h_m$, as above, calculate the quantity flowing by the use of the following formulæ :—

$$\left.\begin{aligned} \text{V (cubic feet per minute)} &= C_1 A \sqrt{h_m}, \\ \text{Q (pounds per minute)} &= C_2 A \sqrt{h_m}, \end{aligned}\right\}^{*} \quad . \quad (10)$$

where $C_1$ and $C_2$ are numerical factors depending upon the temperature and pressure.

* Note that $\sqrt{h_m}$ is obtained directly from § 2 of this summary.

If the temperature $t$ is measured in degrees Fahrenheit and the barometric pressure $b$ in inches of mercury,

$$\left.\begin{aligned} C_1 &= 953\sqrt{\frac{460+t}{b}}, \\ \text{and } C_2 &= 1264\sqrt{\frac{b}{460+t}}, \end{aligned}\right\} \quad . \quad . \quad . \quad (10a)$$

whilst for the Centigrade and millimetres of mercury scales these coefficients become

$$\left.\begin{aligned} C_1 &= 6450\sqrt{\frac{273+t}{b}}, \\ \text{and } C_2 &= 186{\cdot}9\sqrt{\frac{b}{273+t}}, \end{aligned}\right\} \quad . \quad . \quad . \quad (10b)$$

(4) The quantity $b$ may be taken as the observed barometric pressure so long as the static pressure in the pipe does not exceed 8 inches of water.* Otherwise $b$ must be increased or diminished by the static pressure, expressed in inches or millimetres of mercury according to the units in which the barometric height is observed. The conversion factors are given above.

When air measurements have frequently to be made it will probably be found useful, in order to save numerical calculation, to prepare charts from which the coefficients $C_1$ and $C_2$ can be read off directly. A specimen chart, suitable for cases in which a wide range of temperature is to be covered by the observations, is shown in Fig. 23. It will be seen that values of $C_1$ and $C_2$ determined from equations (10$a$) are plotted for values of $b$ of 29, 30, and 31 inches of mercury, so that the value of $C_1$ or $C_2$ for any temperature within the range covered by the chart is given by the ordinate of the appropriate curve. Values for intermediate pressures can be obtained with sufficient accuracy by interpolation, and for pressures over 31 inches or under 29 inches (both of which are of infrequent occurence) by extrapolation.

**Measurement of the Mean Total Head at a Section of a Pipe.**—It was shown in Chapter IV. that the resistance of a length of pipe between any two sections A and B to the flow of air along it may be expressed as the difference between the true mean total heads $T_1$ and $T_2$ at A and B. It was also

* Without causing an error of more than 1 per cent. in V or Q.

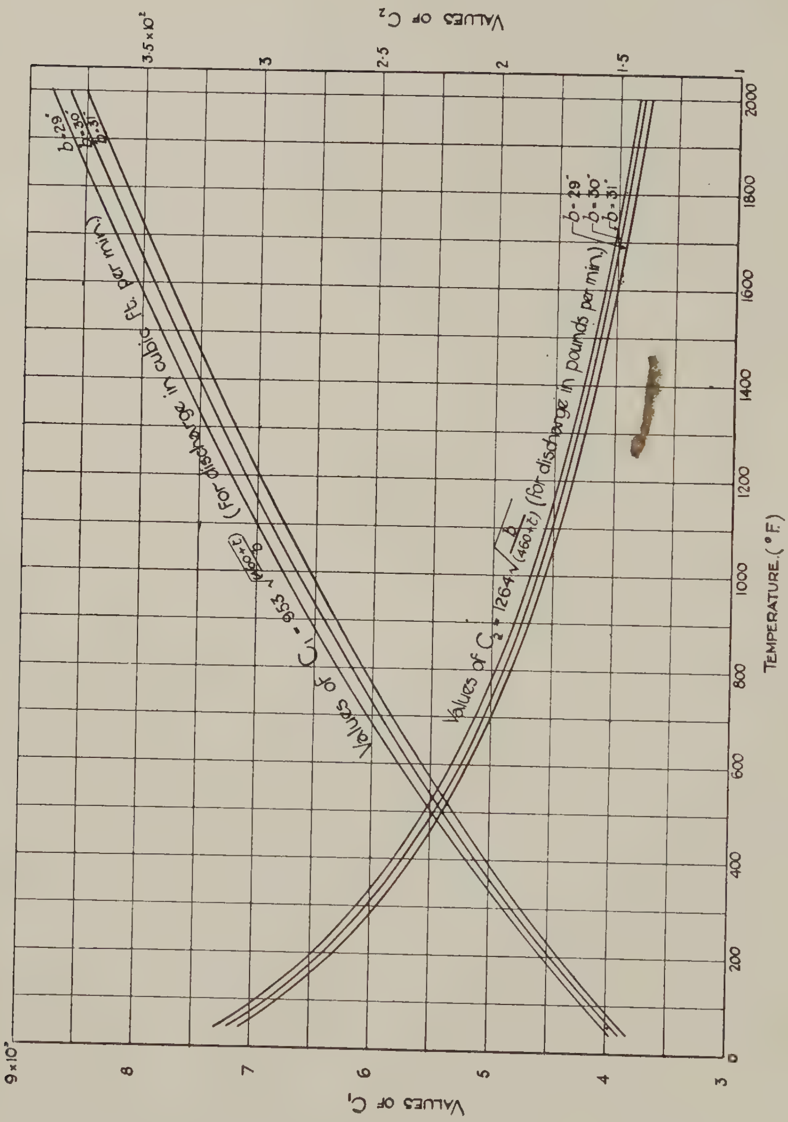

Values of $C_2$
Values of $C_1$
Temperature (°F)
Values of $C_1 = 953\sqrt{\frac{(460+t)}{b}}$ (for discharge in cubic ft. per min.)
Values of $C_2 = 1264\sqrt{\frac{b}{(460+t)}}$ (for discharge in pounds per min.)
b=29"
b=30"
b=31"
9×10²
8
7
6
5
4
3
3.5×10²
3
2.5
2
1.5
1
0
200
400
600
800
1000
1200
1400
1600
1800
2000

FIG. 23.

shown that if $v$ is the velocity at a section of a circular pipe at radius $r$, and R is the internal pipe radius

$$T = \frac{1}{2}\rho \frac{\int_0^R v^3 r dr}{\int_0^R v r dr} + \frac{\int_0^R p v r dr}{\int_0^R v r dr} . \qquad (11)$$

In order, therefore, to determine T the value of each of the integrals in (11) must be calculated. For the highest accuracy $p$ and $v$ should be measured at a sufficient number of radii to enable curves showing the variation of $v$ and $p$ with $r$ to be defined. From these data it is possible to obtain, at each value of $r$, values of the quantities $v^3r$, $vr$, and $pvr$, and to plot them against $r$. The areas enclosed by the curves thus obtained and the axis of $r$ will be equal respectively to

$$\int_0^R v^3 r dr, \quad \int_0^R v r dr, \text{ and } \int_0^R p v r dr.$$

The value of T from (11) is then calculable.

Some care is necessary with regard to the units. Usually $v$ at each point will be determined from Pitot and static observations (in some cases possibly by means of an anemometer), and will be expressed in feet per second. The first term of (11), which we may call V, will thus have the dimensions of $\frac{1}{2}\rho \times (\text{velocity})^2$, i.e. of a pressure in pounds per square foot. In evaluating the second term (which we will denote by P) each value of $v$ in feet per second may be multiplied by the observed value of the static pressure $p$ in inches of water, for the integral in the numerator; when this is divided by the integral in the denominator in which $v$ is in feet per second, the resulting value of P will be in inches of water. Hence for the total head T to be expressed in inches of water, the value of V must be converted to these units, and since a pressure of 5·2 pounds per square foot corresponds very closely to 1 inch of water, we have

$$T = \frac{V}{5 \cdot 2} + P \text{ inches of water.}$$

This method is laborious and usually unnecessarily so. In most cases it will be sufficient to observe $v$ and $p$ at the centres of zones of equal area, just as in determining the mean velocity $v_m$ in order to calculate the volume flowing. As a rule, in fact,

the observations of velocity head taken for the determination of $v_m$, supplemented by measurements of $p$ at the same points, may be used for calculating T. We shall then have a number of velocity heads $h_1$, $h_2$, $h_3$, etc., taken at points chosen in accordance with the rules laid down on page 55, and a number of static pressures $p_1$, $p_2$, $p_3$, etc., at the same points. The simplest procedure will then be to determine the velocities $v_1$, $v_2$, $v_3$, etc., corresponding to these velocity heads. It is then easy to show that

$$T = \frac{1}{5{\cdot}2} \times \tfrac{1}{2}\rho \frac{v_1{}^3 + v_2{}^3 + v_3{}^3 + \dots}{v_1 + v_2 + v_3 + \dots} + \frac{p_1 + p_2 + p_3 + \dots}{n},$$

where $n$ is the total number of static pressure readings taken. It is here assumed that the static pressure does not vary much across the section. If it does the flow is not everywhere parallel to the axis of the pipe. When the departure from axial flow is very pronounced it is doubtful whether any method of measurement will give good accuracy, since both $v$ and $p$ as measured at any point may be in error on account of the inclination of the local direction of flow to the Pitot and static head. An attempt should always be made to avoid the necessity for taking measurements at such sections, but if there is no alternative, the second term in T should be obtained from the expression

$$\frac{p_1 v_1 + p_2 v_2 + p_3 v_3 + \dots}{v_1 + v_2 + v_3 + \dots}$$

instead of that given above.

The modification of these methods to suit pipes of rectangular section follows simply on the lines of the measurement of $v_m$ for such sections (see p. 52) and needs no further comment.

**Practical Notes on the Use of Pitot and Static Tubes.**—In determining the mean rate of flow or the mean total head at a section, we have seen that the Pitot and static tube must be inserted into the pipe through a hole cut in the side, and traversed across the section, readings being taken at a number of pre-determined points. It is convenient to provide a number of wire rings fitting the stem of the instrument fairly tightly, whose positions can be adjusted prior to the insertion of the tube into the pipe, so that when each ring in turn is up against the outside of the pipe, the axis of the head is at one of the desired positions inside. A linear scale marked on the stem further facilitates the correct location of the points of measurement.

The head of the instrument should, of course, point upstream, with its axis parallel to the direction of motion of the air, which is usually assumed to be parallel to the walls of the pipe. A useful method of ensuring that the head is in the desired attitude is to have, permanently fixed to the extreme end of the stem outside the pipe, a straight-edge, five or six inches in length, parallel to the head, which can be sighted along the outside wall of the pipe. The hole cut in the pipe wall should not be larger than is necessary for the introduction of the tubes, and should preferably be, as far as possible, stopped up when the instrument is in place.

Combined Pitot-static tubes of the N.P.L. pattern or of the alternative type shown in Fig. 9 may be used in pipes up to about 15 inches diameter, but in practice it frequently happens either that the length of these tubes is too short to permit them to be traversed completely across a section of a pipe or that the instruments are unsuitable for some other reason. In such cases, when modified forms of Pitot tubes have to be made up to suit particular conditions, it obviously simplifies construction to have the total head and static tubes separate, and to clamp them in juxtaposition. Satisfactory apparatus can be made from material to be found in the ordinary works machine shop: steel or brass tubing of about $\frac{5}{16}$ or $\frac{3}{8}$-inch external diameter is quite suitable for most purposes. Some care is necessary in shaping the heads of both tubes if it is intended to use them clamped together. Although any open-ended tube facing the wind will indicate total head correctly, if the tubing is at all thick-walled the flow of air around the outside will be somewhat disturbed, and may therefore lead to error in the measurement of the static pressure by means of another tube placed near the first. For this reason it is advisable to file the outside of the total head tube near the end to form a taper, so as to present only a thin annular edge to the wind. For the static tube it will probably be found simplest to adopt the type having a hemispherical nose. This tube can easily be constructed by bending a straight piece of tubing to form the head and the stem, and closing the open end of the head by means of a hemispherical plug having a short shank that forms a push fit or screws into the tube, the joint being made airtight by finishing it off with solder. Care should be taken to leave no gap between the plug and the end of the tube: the outline of the head should be smooth and continuous. The static holes should be six tube diameters behind the junction of the

plug and the head, and the stem eight diameters behind the holes, if convenient. In this case the tube will indicate the static pressure correctly, but if, for some reason, it is more convenient to have the stem at some other distance behind the static holes, the correction to be applied to the observed pressure in order to obtain the true static pressure can easily be obtained by reference to Fig. 8. The actual size of the static holes is not of great importance, provided they are not too large; a diameter of about 0·05 inch in $\frac{3}{8}$-inch diameter tubing is quite suitable. It is important that in drilling the holes the outside of the tube should be left entirely free from burrs.

When separate total head and static tubes are used clamped together, the air flow round each will be affected by the presence of the other, so that if the tubes are too near each other the accuracy of the static head reading will be impaired by the interference effect of the total head tube, although the reading of the latter will not be correspondingly affected. Experiments made by the author have shown that the tubes may safely be clamped with their axes not less than 5 tube diameters apart. Similarly, all brackets and clamps should preferably be confined, if possible, to the stems of the tubes outside the pipe, in order that they should not vitiate the static pressure observations by giving rise to disturbances similar to the effect of the stem.

It may be mentioned here that when separate tubes are used for the measurement of total head and static pressure, these observations need not necessarily be made simultaneously provided the conditions of flow are reasonably steady and constant. It is often more convenient to read first the static pressure and then the total head at the point (or *vice versa*), the appropriate tubes being successively introduced into the pipe. This necessitates the cutting of smaller holes in the piping than when the tubes are inserted clamped together. The velocity head is then, of course, obtained by subtracting the observed reading of the static tube from that of the total head tube, due attention being paid to the signs of the observed pressures (i.e. whether they are above or below the atmospheric pressure).

**Readings of Fluctuating Pressures.**—Serious difficulty is frequently experienced in Pitot-static observations on account of the unsteadiness of the air stream. In such cases the liquid in the manometer oscillates, at times over a large range, and it is not easy to estimate the mean value of the pressure to

the necessary degree of accuracy. A method of overcoming this difficulty has been suggested by Mr. L. F. G. Simmons of the National Physical Laboratory. It depends in principle on the fact that a fluctuating pressure acting on one end of a long tube is not transmitted instantaneously to the other end. Moreover, the amplitude of the transmitted fluctuations is not the same as that of the applied fluctuations. A tube of given dimensions exerts, in effect, a definite amount of damping on the motion of a pressure wave along its length. Now the amount of damping contributed by the total head tube will obviously be less than that of the static tube on account of the larger resistance of the small static orifices. Hence, even if the same fluctuating pressure acts both at the mouth of the total head tube and at the static holes, the fluctuations will not be damped out to the same extent by the time the pressures are transmitted to the manometer, unless extra damping is

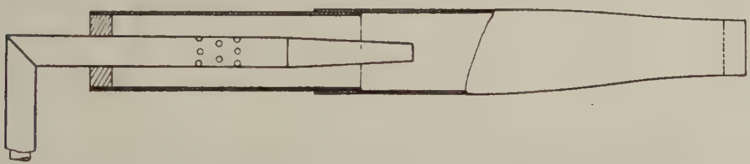


FIG. 24.—Method of equalising the damping on both sides of a Pitot-static tube.

introduced in the total head connecting tube. It appears, therefore, that the total head connection must be made longer than the static connection.

The necessary lengths of tubing can readily be determined by the method illustrated in Fig. 24. The head of the Pitot-static tube is introduced through a cork into a length of brass tubing about $\frac{3}{4}$ inch in diameter, the other end of which is connected, as shown, to a convenient length of large-bore rubber tubing closed by a second cork. By squeezing this rubber tubing in the hand, fluctuating pressures can be applied simultaneously to the mouth of the total head tube and the orifices in the static tube. If the other ends of these tubes are connected to the manometer in the usual manner, the lengths of the connecting tubes can be adjusted until the oscillations of the manometric liquid become quite small even for large applied pressure fluctuations. When the required lengths of tubing have been thus ascertained by trial and error, it will be found that even under unsteady conditions of measurement

sufficiently steady readings will generally be obtained on the manometer.

On the assumption that the same fluctuations of pressure act on both the total head and static orifices, and that this pressure oscillates symmetrically about a mean value, the resultant reading on the manometer will be the velocity head corresponding to the true mean rate of flow at the position occupied by the Pitot-static tube. Actually, these conditions will not be completely realised. The total head fluctuations will be a combination of velocity and static pressure variations, whereas the static orifices will be affected only by variations of static pressure. Hence a system balanced in the above manner under pressure variations only, will not give entirely steady readings when used to measure flow in a pipe where the velocity and static pressure are both fluctuating. It will nevertheless usually be found that greatly improved steadiness can be secured by adopting the procedure described above.

# CHAPTER VI.

## THE PLATE ORIFICE, VENTURI TUBE, AND SHAPED NOZZLE.

The Pitot tube, in conjunction with a properly designed static tube, has the important advantage that its calibration has been definitely established by careful experiment, so that it can be, and, in fact, is, regarded as a fundamental standard for the measurement of wind speed. In the absence of any other standard of equal precision, it must play an important part in the calibration of all other instruments for air speed measurements. It is, however, subject to certain shortcomings for practical work. In the first place, as we have seen, it is not well suited to a rapid determination of quantity since the pipe section has to be traversed, and a certain amount of arithmetical work has to be performed. Secondly, the pressures to be observed are very often small so that sensitive manometers are required to give the necessary accuracy, and the design of such instruments to read accurately the velocity heads corresponding to air speeds of less than 1000 feet per minute, presents serious practical problems.

For these reasons methods have been sought in which the pressures to be measured are automatically augmented, and in which single readings enable the mean velocity in a pipe to be determined. Although such methods lack the certainty of the Pitot tube when used with a manometer of the requisite precision—as a general rule, in fact, it is safest to calibrate the apparatus employed by means of the Pitot tube if the highest accuracy is required—they are undeniably superior to this instrument as regards the ease and rapidity with which the necessary observations can be performed. Briefly, these methods consist in introducing into the stream some device such as a nozzle or an orifice plate which, at a given section of the pipe, constricts the free area through which the air can pass and so effects a local increase in velocity. It is then found that a comparatively large drop of static pressure is

experienced by the air in passing from the full diameter of the pipe to the section at which the constriction, and hence also the velocity, is a maximum, and that this pressure drop can be used as a measure of the air speed.

**Equations for the Flow through Orifices and Nozzles.**—Suppose that in a given pipe the walls are shaped as shown in Fig. 25, so that the area at section BB is less than at AA. Let $p_1$, $v_1$, $\rho_1$, $a_1$ be respectively the absolute static pressure (measured from zero), the mean velocity, the density of the air, and the area of the section at AA, and $p_2$, $v_2$, $\rho_2$, $a_2$ be the corresponding quantities at BB. Also imagine that the pipe walls at AA and BB and the directions of flow at these sections are parallel to the axis of the pipe.

We shall assume the flow to be frictionless, and at a later stage introduce numerical coefficients to cover the error involved

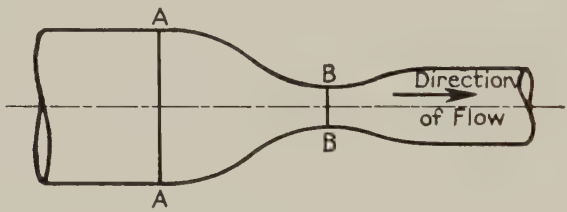



FIG. 25.

by this and other assumptions. Such coefficients will have to be determined experimentally.

The difference between the static pressures $p_1$ and $p_2$ will depend mainly on the ratio of $a_1$ to $a_2$. In many cases the ratio $\frac{p_1}{p_2}$ does not differ much from unity,* and in these circumstances $\rho_1$ and $\rho_2$, the densities of the air at the two sections, may be considered as effectively equal.

We start with the general equation derived in Chapter II., page 10, viz.,

$$\frac{v^2}{2} + \int \frac{dp}{\rho} = \text{constant},$$

* It must be remembered that $p_1$ and $p_2$ are measured from zero, so that their mean value in most cases will be approximately atmospheric pressure or, say, 400 inches of water. Hence in such cases they may differ by about 8 inches of water—an easily measurable amount—before the ratio of one to the other differs from unity by more than 2 per cent.

and apply it to the air between sections AA and BB. We then have

$$\frac{v_2^2 - v_1^2}{2} = \int_{p_2}^{p_1} \frac{dp}{\rho} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

**Case** $(a)$ : $\frac{p_2}{p_1}$ **Nearly Equal to Unity**.—In this case $\rho_2$ may be taken as equal to $\rho_1$, which is equivalent to regarding the air as incompressible. Equation (1) becomes

$$\frac{v_2^2 - v_1^2}{2} = \frac{p_1 - p_2}{\rho}. \quad . \quad . \quad . \quad (2)$$

Also, the weight of air flowing per unit time across AA is equal to that across BB, i.e.

$$\rho_1 a_1 v_1 = \rho_2 a_2 v_2,$$

or

$$v_1 = \frac{a_2 v_2}{a_1}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Substituting (3) in (2) we have

$$v_2^2 = \frac{2(p_1 - p_2)}{\rho\left(1 - \frac{a_2^2}{a_1^2}\right)}, \quad . \quad . \quad . \quad . \quad (3a)$$

and the weight of air flowing per second * is given by

$$q = g\rho a_2 v_2 = a_2 g \sqrt{\frac{2\rho(p_1 - p_2)}{1 - \frac{a_2^2}{a_1^2}}}. \quad . \quad . \quad (4)$$

Actually, the flow will be less than this, the true value of $q$ bearing a ratio $\alpha$ to the theoretical value, $\alpha$ being less than unity; $\alpha$ is known as the coefficient of discharge and has to be determined, as stated above, by experiment. Inserting $\alpha$ in (4), and writing $r$ for the ratio of the upstream and downstream areas $\frac{a_1}{a_2}$, we have finally

$$q = \alpha a_2 g \sqrt{\frac{2\rho r^2(p_1 - p_2)}{r^2 - 1}}, \quad . \quad . \quad (5)$$

* It should be remembered that $\rho$ here has its customary significance of $\frac{\text{weight per unit volume}}{g}$ so that the weight of a volume V of air is $g\rho$ V units of weight.

which is the equation for the flow of an incompressible fluid through an orifice or nozzle.

**Case** (*b*) **:** $\frac{p_2}{p_1}$ **Appreciably Less than Unity.**—Compressibility can no longer be neglected, so that $\rho_2$ cannot now be taken as equal to $\rho_1$; the density will now change with $p$ according to the usual laws for gases. It is almost always the case in practice that the flow along the pipe is sufficiently rapid for the expansion ($p_2$ is less than $p_1$) in the nozzle or orifice to be taken as adiabatic, so that we have

$$\frac{p_1}{\rho_1^{\gamma}} = C = \frac{p_2}{\rho_2^{\gamma}}, \quad . \quad . \quad . \quad . \quad (6)$$

where C is a constant and $\gamma$ is the usual index for adiabatic expansion (1·408 for dry air).

Hence in this case (1) becomes, since, from (6), $\frac{1}{\rho} = \left(\frac{C}{p}\right)^{\frac{1}{\gamma}}$,

$$\frac{v_2^2 - v_1^2}{2} = C^{\frac{1}{\gamma}}\int_{p_2}^{p_1}\frac{dp}{p^{\frac{1}{\gamma}}} = C^{\frac{1}{\gamma}}\frac{\gamma}{\gamma - 1}\left\{p_1^{\frac{\gamma-1}{\gamma}} - p_2^{\frac{\gamma-1}{\gamma}}\right\},$$

and, if we put $C^{\frac{1}{\gamma}} = \frac{p_1^{\frac{1}{\gamma}}}{\rho_1}$, this reduces to

$$\frac{v_2^2 - v_1^2}{2} = \frac{\gamma}{\gamma - 1} \cdot \frac{p_1}{\rho_1}\left\{1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right\} \quad . \quad . \quad (7)$$

The equation of continuity of flow, i.e. equal weights of air passing sections AA and BB in unit time, is, as before,

$$\rho_1 a_1 v_1 = \rho_2 a_2 v_2,$$

or, if the value for $\frac{\rho_2}{\rho_1}$ from (6) is inserted,

$$v_1 = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}\frac{a_2}{a_1}v_2.$$

Putting this value of $v_1$ in (7), and reducing, we have

$$v_2 = \sqrt{\frac{2 \cdot \frac{\gamma}{\gamma - 1} \cdot \frac{p_1}{\rho_1}\left\{1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right\}}{1 - \frac{a_2^2}{a_1^2}\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}}}}, \quad . \quad . \quad . \quad (7a)$$

so that, inserting the discharge coefficient $\alpha$, and putting, as before, $r = \frac{a_1}{a_2}$, the weight of air flowing per second becomes

$$q = \alpha a_2 g \rho_2 v_2 = \alpha a_2 g \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} \rho_1 v_2$$

$$= \alpha a_2 g \sqrt{\frac{2r^2 \cdot \frac{\gamma}{\gamma - 1} \cdot \rho_1 p_2^{\frac{2}{\gamma}} p_1^{\frac{\gamma - 2}{\gamma}} \left\{1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}\right\}}{r^2 - \left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}}}}$$

$$= \alpha a_2 g \sqrt{\frac{2r^2 \frac{\gamma}{\gamma - 1} p_1 \rho_1 \left\{1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}\right\} \left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}}}{r^2 - \left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}}}} \quad . \quad (8)$$

This equation can be thrown into a form which is more convenient for practical purposes, since it involves the pressure drop $p_1 - p_2$ across the nozzle or orifice, which is easily measurable. Moreover, the modified form of the equation is more readily comparable with (5), the equation for the flow of an incompressible fluid.

Let us write R for the expansion ratio $\frac{p_2}{p_1}$ wherever it occurs in (8); then we may make the following substitution for $p_1$ where it occurs by itself $\left(\text{not as part of the ratio } \frac{p_2}{p_1}\right)$:—

$$p_1 = \frac{p_1}{(p_1 - p_2)} \times (p_1 - p_2) = (p_1 - p_2) \times \frac{1}{1 - R}.$$

Equation (8) may now be written thus:—

$$q = \alpha a_2 g \sqrt{\frac{2\rho_1 r^2 (p_1 - p_2)}{r^2 - 1}} \times$$

$$\sqrt{\frac{(r^2 - 1) \frac{\gamma}{\gamma - 1} \cdot \frac{1}{1 - R} \cdot (1 - R^{\frac{\gamma - 1}{\gamma}}) R^{\frac{2}{\gamma}}}{r^2 - R^{\frac{2}{\gamma}}}} \quad . \quad . \quad (9)$$

Comparing this equation with (5), we see that it is the equation for the discharge of air treated as an incompressible

fluid, modified by the term under the second radical, which allows for the effect of compressibility. Equation (9) is the general equation for the flow of a fluid through a constriction. In practice the constriction in a pipe line consists of some device such as an orifice plate, a nozzle, or a Venturi tube, each of which is considered in detail in the sequel. When these appliances are suitably proportioned to the flow, a pressure difference ($p_1 - p_2$ in equation (9)) is set up of such a magnitude that it can easily be measured to the necessary degree of accuracy (within 1 per cent.) without a manometer of high sensitivity.

It will be seen that this pressure difference depends upon the ratio of the diameter of the pipe to that of the minimum section presented by the constriction, an increase in this ratio producing a corresponding increase in the pressure difference to be measured. Practical limitations may of course prevent the minimum section from being too small in relation to the pipe, but even so the pressure difference can be made much larger than the velocity head in the open pipe. As an example of the advantage to be derived from this feature of a constriction we may take the case of a 6-inch diameter pipe in which the mean velocity is 20 feet per second. The corresponding velocity head is about 0·09 inch of water, whereas if a 3-inch diameter plate orifice be inserted in the pipe, the pressure drop across it will be about 3·7 inches of water, i.e. about 40 times the velocity head.

A further merit of the method of measuring the flow in a pipe by means of a constriction lies in the fact that, since no traverse of the pipe is necessary, as in the case of Pitot tube measurements, it lends itself readily to the taking of continuous records of the quantity of air flowing. For this purpose it is merely necessary to transmit the upstream and downstream pressures to a recording differential pressure gauge, the charts for which may be graduated to read in inches of water, or, if great accuracy is not of importance, directly in pounds of air per minute. In the latter case, of course, the corrections for compressibility and the variations of density of the air with temperature and pressure have to be neglected, and, in order to reduce errors as far as possible, it is advisable to adjust the gauge to read correctly at the estimated mean conditions of temperature, pressure, and pressure drop across the orifice to which it is likely to be subjected when in use.

The chief difficulty in the practical application of this

method of measurement arises in assigning a value to $\alpha$, the coefficient of discharge. A large amount of experimental work has been done, particularly with the thin plate orifice, to establish values of $\alpha$ for the various forms of constricting devices, but many of the experimenters do not seem to have been fully acquainted with the various factors which affect the value of the coefficient, and their work has in consequence lost some of its value, as no general conclusions can be drawn therefrom.

Before we discuss practical cases in detail it will be instructive to consider the question of the coefficient of a constriction from a general point of view. In a device such as a properly designed Venturi tube or a shaped nozzle in which the stream at efflux is parallel, the coefficient approaches the value unity, so that the actual discharge is only slightly less than the theoretical. The slight reduction that does occur is mainly attributable to the frictional loss in the passage of the fluid through the constriction. In the case of an orifice cut in a thin plate inserted in the stream, the actual discharge is considerably less than that calculated by the use of equation (9), taking $a_2$ as the area of the orifice. This is due to the fact that the stream issues from the orifice not in the form of a parallel jet, but as a convergent jet, the section of minimum area, which is of the order of 0·6 or 0·7 times the area of the orifice, occurring at a distance of somewhat less than a pipe diameter or so from the plane of the orifice. The section of minimum area is known as the *vena contracta* ; after passing this section the jet commences to expand, and the pipe will again run full some few diameters downstream from the orifice. Further reference to orifice coefficients will be found below.

**Variables Affecting the Discharge Coefficient.**—Let us now consider the variables that affect the coefficient of discharge of a constricting device in the general case of the flow of a compressible fluid. These variables will be found to be :—

$\left.\begin{array}{l}\text{(}a\text{) the density of the fluid}\\ \text{(}b\text{) the viscosity of the fluid}\end{array}\right\}$ at the section of minimum area of the constriction ;

(*c*) a speed characteristic of the flow ;

(*d*) a linear dimension characteristic of the size and shape of the constriction ;

(*e*) a property of the fluid characteristic of its compressibility ;

(*f*) the ratio of the diameter of the constriction to that of the pipe.

For (*c*) we may take $v_2$, the mean speed of the fluid through the throat, or section of minimum area of the constriction, and for (*d*) the diameter $d_2$ of the throat. For the compressibility it is customary to take the ratio $\frac{v_2}{V}$ where V is the speed of sound in the fluid. This ratio, which can be shown to be characteristic of the compressibility, can be reduced to the form $\sqrt{\frac{\rho_2 v_2^2}{\gamma p_2}}$. By making use of the relation between $v_1$ and $v_2$, and between $\frac{p_1}{\rho_1^\gamma}$ and $\frac{p_2}{\rho_2^\gamma}$, we can transform the expression $\sqrt{\frac{\rho_2 v_2^2}{\gamma p_2}}$ to

$$\sqrt{\frac{2\left\{\left(\frac{p_2}{p_1}\right)^{\frac{1-\gamma}{\gamma}} - 1\right\}}{(\gamma - 1)\left\{1 - \frac{a_2^2}{a_1^2}\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}}\right\}}}.$$

The important point to notice is that this expression is a function of the expansion ratio $\frac{p_2}{p_1}$ and may be denoted by $(\beta)\frac{p_2}{p_1}$.

The variables mentioned above may be collected into three groups, each of which may be regarded as a single variable upon which the value of $\alpha$ in part depends. These new variables will be found to be $\frac{v_2 d_2 \rho_2}{\mu_2}$, $(\beta)\frac{p_2}{p_1}$, and $\frac{d_2}{d_1}$, the first of which, when $\nu$ is substituted for $\frac{\mu}{\rho}$, becomes $\frac{v_2 d_2}{\nu_2}$; $\alpha$ may therefore be represented by the equation,

$$\alpha = f\left(\frac{v_2 d_2}{\nu_2}, (\beta)\frac{p_2}{p_1}, \frac{d_2}{d_1}\right), \quad . \quad . \quad . \quad (10)$$

where $f$ is some function of the three variables concerned

This is an important equation which merits careful study, and we may profitably consider its practical significance. If we have two geometrically similar * constrictions of different

* Two bodies, A and B, are geometrically similar when all linear dimensions of A bear a constant ratio to the corresponding dimensions of B.

sizes, the ratio $\frac{d_2}{d_1}$ for each will be the same. If, further, we arrange the conditions of flow so that $(\beta)\frac{p_2}{p_1}$ and $\frac{v_2 d_2}{\nu_2}$ are the same for the two constrictions, equation (10) indicates that the coefficients of discharge for the two constrictions will be equal, despite the fact that the latter are of different sizes and that the individual values of $p_1$, $p_2$, $v_2$, etc., may not be the same in the two cases. If any one of the quantities in equation (10) alters in such a manner as to change the value of the ratio in which it occurs, the coefficient $\alpha$ is liable to change.

In the case of the flow of an incompressible fluid, the term $(\beta)\frac{p_2}{p_1}$ ceases to have an influence on the type of flow through the constriction, and hence, on the value of $\alpha$. Further, for a series of geometrically similar constrictions, $\frac{d_2}{d_1}$ will have the same value, and the value of $\alpha$ for this series can be changed only by a change in the value of $\frac{v_2 d_2}{\nu_2}$. Hence for the flow of an incompressible fluid through any one of a series of geometrically similar constrictions, equation (10) reduces to

$$\alpha = f\left(\frac{v_2 d_2}{\nu_2}\right), \qquad . \quad . \quad . \quad (11)$$

where the form of the function $f$ is now the same for all members of the series, but will vary from one series to another, and $\nu$ may be measured anywhere in the pipe, since it does not vary with pressure.

From (11) it is apparent that if any one of the quantities in the ratio $\frac{v_2 d_2}{\nu_2}$ is altered, the value of $\alpha$ will not change if one or both the other quantities are correspondingly altered by amounts such that the ratio remains constant. Thus, suppose for a given form of constriction a value of $\alpha$ is obtained for water flowing at a mean speed $v_2$ through the section of minimum area. The value of $\nu$ for air is about 13 times that of water, so that the given constriction will have the same value of $\alpha$ with air, if the speed is increased thirteen-fold. Alternatively, a geometrically similar constriction, having a throat diameter thirteen times that of the original, will have a value of $\alpha$ for

air equal to that of the original for water, the throat speeds in the two cases being the same.*

Although equations (10) and (11) do not provide a means of obtaining actual numerical values of $\alpha$, they are of value in that they indicate the manner in which this coefficient will vary under different conditions. They therefore furnish a basis upon which the work of various experimenters can be correlated so that their results can be given the widest possible application.

We turn now to a consideration of the various practical methods employed to introduce a constriction in a pipe line in order to measure the flow from the pressure drop set up.

**The Plate Orifice.**—Mechanically, the plate orifice is the simplest of the devices under consideration in this chapter, and is, moreover, the one most easily inserted into an existing pipe line with a minimum of alteration to the lay-out. Unfortunately, however, it is the device in regard to whose discharge coefficient the most uncertainty exists. As usually employed it consists simply of a thin plate or diaphragm, clamped between two flanges of the pipe, and having a circular hole, coaxial with the pipe, cut in it, as in Fig. 26. Two pressure taps, one upstream and one downstream of the plane of the plate, serve as a means of measuring the static pressure drop $(p_1 - p_2)$ from which the delivery is calculated according to equation (9).

Equation (9) can be thrown into a form which is more convenient for practical use. As it stands it will give the quantity of air flowing in pounds per second, the area $a_2$ being measured in square feet, $\rho_1$ in slugs per cubic foot, and $p_1$ and $p_2$ in pounds per square foot. In practice it is usually more convenient to derive the quantity Q in pounds per minute, and to measure the area $a_2$ of the orifice in square inches, and the pressure difference $p_1 - p_2$ in inches of water. It will also be convenient to insert the value for $\rho$ under standard conditions of temperature and pressure (0·00237 at 29·92 inches of mercury and at 60° F.), and to apply a correction for the conditions prevailing at the time of the observations. The procedure in modifying equation (9) is similar to that performed in detail in equations (3), (4), and (5) of Chapter V., and need not

* In this illustration it is assumed that $\frac{p_2}{p_1}$ is nearly unity, so that the air can be regarded as incompressible.

be repeated here. The final form of the modified equation as recommended for practical use will be found to be, if we write C for the compressibility factor represented by the second radical term in equation (9),

$$Q = 8{\cdot}78aa_2\sqrt{\frac{r^2}{r^2 - 1} \cdot (h_1 - h_2)\,\frac{b}{(460 + t)}} \times C$$

pounds per minute, . . . . (12)

where $a_2$, as stated above, is the area of the orifice in square inches, $(h_1 - h_2)$ is the pressure drop across the orifice in

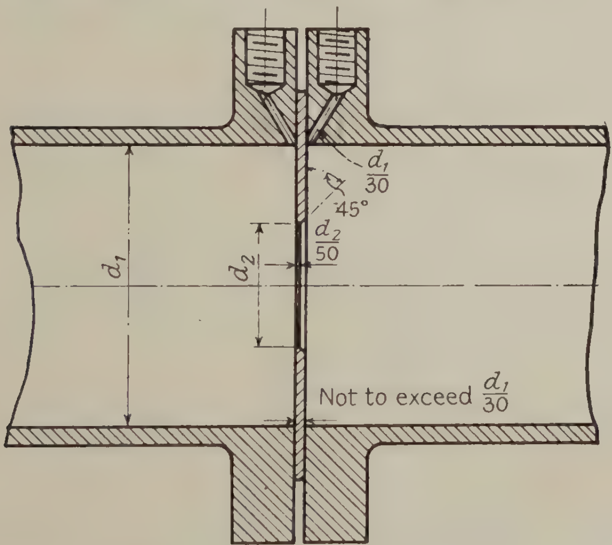



FIG. 26.—Hodgson's sharp-edged orifice.

inches of water, $t$ is the temperature of the air in the throat of the orifice in degrees Fahrenheit, and $b$ is the barometric height in inches of mercury. If $h_1$, the static pressure head in the pipe upstream of the orifice, is more than 8 inches of water above or below atmospheric pressure, $b$ must be increased or diminished by the equivalent of $h_1$ in inches of mercury, according to the note on page 58.

We have now to consider the value of the discharge coefficient $\alpha$, and the magnitude of the compressibility factor C. As we have already observed, the stream immediately on leaving the orifice is convergent, and reaches a minimum

area, appreciably less than that of the orifice, some distance downstream of the orifice. It is not until this section is reached that the flow is parallel to the axis of the orifice. Now the equations hitherto derived for flow through constrictions are based upon the assumption that the flow is parallel at the sections of maximum and minimum area. Strictly, therefore, in applying these equations to orifice flow, the area $a_2$, wherever it occurs, should be taken as the area of the vena contracta, and not as that of the orifice itself. Similarly, the downstream pressure $p_2$ should be the static pressure at the vena contracta.

Obviously, the area of the vena contracta cannot be measured accurately in practice, nor can its exact position be easily determined; it is therefore usual to take $a_2$ as the area of the orifice, and to make the experimentally determined discharge coefficient suit this condition. It is because $a_2$ is appreciably greater than the area of the vena contracta that the discharge coefficient is so much less than unity. Approximate measurements of the vena contracta have been obtained from experiments made with orifice plates in glass tubes along which water was flowing. It is interesting to note that when this area was substituted for $a_2$ in equation (5), the value of $\alpha$ was found to be very much nearer unity, and of the same order as that for a Venturi tube or shaped nozzle, in which the minimum section corresponds with the actual minimum area through which flow takes place. The pressure at the vena contracta is found to be less, but not much less, than that close to the downstream side of the orifice, whilst on the upstream side the pressure rises somewhat in the immediate vicinity of the orifice plate.

**Coefficients of Orifices.**—In view of these variations of pressure along the pipe in the neighbourhood of the orifice, and of the fact that the pressure distribution, even if accurately determined for one set of conditions would not necessarily hold for others, it appears advisable always to measure the pressures as closely as possible to the upstream and downstream planes of the orifice plate, and this course has been adopted by J. L. Hodgson in England and by Jakob and Kretschmer in Germany. The researches of Hodgson * are of great practical value. Recognising the amount of variation that was to be

* *Proc. Inst. C.E.*, vol. cciv., 1916-17, Part II., and vol. ccx., p. 272. See also *Trans. Inst. Naval Architects*, vol. lxiv., 1922, p. 189; *Engineering*, 7th March, 1924, p. 314; *Inst. Civil Engineers, Selected Engineering Papers*, No. 31, 1925.

expected in $\alpha$ with change of shape of the orifice, he concentrated on the form of sharp-edged orifice shown in Fig. 26, which is reproduced from *Engineering* for 7th March, 1924. The pressure connections will be seen to be close to the planes of the plate, and the general proportions of the orifice are as shown in the figure.

Hodgson determined the coefficients of such orifices under various conditions and with a variety of compressible and incompressible fluids, and found that certain general conclusions could be drawn. In discussing these results we may refer to equation (10), which indicates the variables upon which $\alpha$ may be expected to depend. The effect of one variable alone may be determined by experiments in which the other two are kept constant. In the first place, Hodgson found that, when testing for the variation of $\alpha$ with $\frac{v_2 d_2}{\nu_2}$ at constant values of $\frac{d_2}{d_1}$ and $(\beta)\frac{p_2}{p_1}$, a wide variation of $\frac{v_2 d_2}{\nu_2}$ was possible without causing a change in $\alpha$. He states that the curve of $\alpha$ against $\frac{v_2 d_2}{\nu_2}$, when the other variables do not change, is a horizontal straight line within the range of values of $\frac{v_2 d_2}{\nu_2}$ of about 16,000 to 1,600,000, which, fortunately, covers the very large majority of cases that are likely to occur in practice. It appears, therefore, that the variation of $\alpha$ with $\frac{v_2 d_2}{\nu_2}$ can nearly always be neglected. Exceptions occur for the extreme cases of slow flow through small orifices or rapid flow through large orifices.

For all except abnormal conditions, it is necessary, therefore, only to consider the effect of changes in $(\beta)\frac{p_2}{p_1}$ and $\frac{d_2}{d_1}$, and, if we confine our attention to air flow, a further simplification is possible, since the variable $(\beta)\frac{p_2}{p_1}$, which is a function of both $\gamma$ and the expansion ratio, can be replaced by the variable $\frac{p_2}{p_1}$. The complete variable $(\beta)\frac{p_2}{p_1}$ need only be considered in comparisons of values of $\alpha$ for different gases; for any one gas $\gamma$ is constant and may therefore be neglected.

Thus, for air flows within the range of $\frac{v_2 d_2}{\nu_2}$ of 16,000 to

1,600,000, the discharge coefficient for Hodgson's orifices would appear to depend only upon $\frac{p_2}{p_1}$ and $\frac{d_2}{d_1}$. The variation with $\frac{p_2}{p_1}$ for constant values of $\frac{d_2}{d_1}$ is stated by Hodgson to be approximately linear; he found further, that at a constant value of $\frac{p_2}{p_1}$, $\alpha$ had a constant value for all values of $\frac{d_2}{d_1}$ less than 0·7, so that provided $\frac{d_2}{d_1}$ is always made less than 0·7 its influence in modifying the value of $\alpha$ may be neglected, and only one variable, viz. $\frac{p_2}{p_1}$, remains to be considered for nearly all cases of air flow. Hodgson found that the variation of $\alpha$ with $\frac{p_2}{p_1}$ was approximately expressible in the form

$$\alpha = 0{\cdot}914 - 0{\cdot}306\frac{p_2}{p_1}. \qquad (13)$$

In a large number of cases $\frac{p_2}{p_1}$ will be found to be between 0·98 and 1·00, and for these conditions $\alpha$ may be taken as 0·61.

Similar results for values of $\frac{p_2}{p_1}$ near unity have been obtained for sharp-edged orifices very similar to those of Hodgson by Jakob and Kretschmer.* Here again the upstream and downstream pressures were measured close to the orifice plate. These tests covered a range of $\frac{v_1 d_1}{\nu_1}$ of 20,000 to 500,00C, where $v_1$ was here taken as the mean velocity in the pipe in which the orifices were installed, and $d_1$ and $\nu_1$ were respectively the diameter of the pipe and the kinematic viscosity of the air at the upstream pressure tap where the pressure was $p_1$. Jakob and Kretschmer's discharge coefficient is equivalent to $\alpha\sqrt{\frac{r^2}{r^2 - 1}}$, and their results are given in the following table, which also includes, for comparison, Hodgson's coefficient 0·61 multiplied by the term $\sqrt{\frac{r^2}{r^2 - 1}}$. Hodgson's results, according to his ex-

* *Regeln für Leistungsversuche an Ventilatoren und Kompressoren,* 1925, p. 16, published by the Verein Deutscher Ingenieure, Berlin.

periments, should only hold for values of $\frac{d_2}{d_1}$ less than 0·7, i.e. for values of $r$ greater than 2, but it is interesting to note that the remarkable agreement between his and the German values continues up to a value of $r = 1{\cdot}33\ \left(\frac{d_2}{d_1} = 0{\cdot}866\right)$, if the coefficient 0·61 is assumed to be valid up to this value. It would therefore appear that a coefficient of 0·61 can be taken with some degree of confidence to apply to sharp-edged orifices of the type shown in Fig. 26 for values of $\frac{p_2}{p_1}$ between 1 and 0·98, and for values of $\frac{d_2}{d_1}$ at least up to 0·7, and possibly up to 0·85. For other values of $\frac{p_2}{p_1}$ an approximate value of $\alpha$ is given by equation (13). As has already been pointed out, however, most practical cases will come within the conditions where $\alpha$ may be taken as 0·61 without correction.

TABLE II.

| $\frac{d_2}{d_1}$. | $r = \left(\frac{d_1}{d_2}\right)^2$. | Jakob and Kretschmer's Discharge Coefficient. | $0{\cdot}61\sqrt{\frac{r^2}{r^2-1}}$. Discharge Coefficient Reduced from Hodgson's Value for $\frac{p_2}{p_1}$ nearly Unity and $\frac{d_2}{d_1} < 0{\cdot}7$. |
|---|---|---|---|
| 0·387 | 6·67 | 0·61 | 0·62 |
| 0·448 | 5·00 | 0·62 | 0·62 |
| 0·500 | 4·00 | 0·63 | 0·63 |
| 0·548 | 3·33 | 0·64 | 0·64 |
| 0·591 | 2·86 | 0·65 | 0·65 |
| 0·633 | 2·50 | 0·66 | 0·67 |
| 0·671 | 2·22 | 0·68 | 0·68 |
| 0·707 | 2·00 | 0·70 | 0·70 |
| 0·742 | 1·82 | 0·73 | 0·73 |
| 0·775 | 1·67 | 0·76 | 0·76 |
| 0·806 | 1·54 | 0·80 | 0·80 |
| 0·837 | 1·43 | 0·85 | 0·85 |
| 0·866 | 1·33 | 0·91 | 0·92 |

**The Compressibility Correction.**—We must consider now the application of the compressibility correction, i.e. the value

of C in equation (12), for values of $\frac{p_2}{p_1}$ differing appreciably from unity. The following table gives values of C for air for values of $\frac{p_2}{p_1}$ between 1·0 and 0·80, and of $r$ from 1·4 to 6·0 By plotting the figures given, the value of C within the ranges of $\frac{p_2}{p_1}$ and $r$ stated can be found for any conditions. Cases outside these ranges will seldom occur. It will be seen that C is sensitive to relatively small changes in $r$ when $r$ is small, but that, at a given value of $\frac{p_2}{p_1}$, the variation of C with $r$ becomes progressively smaller as $r$ increases. For example, when $\frac{p_2}{p_1}=0·80$, the value of C changes by a little over 1 per cent. as $r$ increases from 3·0 to 6·0, whereas for the limiting case when $r$ is infinite, which corresponds approximately to the condition of air discharging from a large chamber through a small orifice, the value of C is within about 0·5 per cent. of its value when $r = 6·0$. The larger the ratio $\frac{p_2}{p_1}$, the smaller is the variation of C over the range of $r$.

TABLE III.

| $r$. | $\frac{d_2}{d_1}$. | Values of C when $\frac{p_2}{p_1}=$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0·98. | 0·96. | 0·94. | 0·92. | 0·90. | 0·88. | 0·86. | 0·84. | 0·82. | 0·80. |
| 1·4 | 0·845 | 0·975 | 0·952 | 0·928 | 0·901 | 0·884 | 0·863 | 0·841 | 0·823 | 0·802 | 0·783 |
| 1·6 | 0·791 | 0·980 | 0·963 | 0·943 | 0·925 | 0·906 | 0·889 | 0·871 | 0·854 | 0·835 | 0·819 |
| 1·8 | 0·745 | 0·982 | 0·968 | 0·950 | 0·934 | 0·918 | 0·902 | 0·886 | 0·871 | 0·854 | 0·838 |
| 2·0 | 0·707 | 0·984 | 0·971 | 0·954 | 0·940 | 0·925 | 0·910 | 0·894 | 0·882 | 0·865 | 0·850 |
| 2·2 | 0·674 | 0·985 | 0·973 | 0·958 | 0·943 | 0·929 | 0·915 | 0·901 | 0·888 | 0·872 | 0·857 |
| 2·4 | 0·646 | 0·986 | 0·974 | 0·960 | 0·946 | 0·932 | 0·919 | 0·904 | 0·892 | 0·877 | 0·863 |
| 2·6 | 0·620 | 0·986 | 0·975 | 0·961 | 0·948 | 0·934 | 0·921 | 0·908 | 0·896 | 0·881 | 0·867 |
| 2·8 | 0·598 | 0·987 | 0·976 | 0·962 | 0·949 | 0·936 | 0·924 | 0·910 | 0·898 | 0·884 | 0·870 |
| 3·0 | 0·577 | 0·987 | 0·976 | 0·963 | 0·951 | 0·938 | 0·925 | 0·912 | 0·899 | 0·885 | 0·872 |
| 3·5 | 0·535 | 0·988 | 0·978 | 0·965 | 0·952 | 0·940 | 0·927 | 0·915 | 0·903 | 0·889 | 0·876 |
| 4·0 | 0·500 | 0·988 | 0·978 | 0·965 | 0·954 | 0·942 | 0·929 | 0·917 | 0·906 | 0·892 | 0·879 |
| 4·5 | 0·471 | 0·988 | 0·978 | 0·966 | 0·954 | 0·942 | 0·931 | 0·919 | 0·907 | 0·893 | 0·881 |
| 5·0 | 0·448 | 0·988 | 0·979 | 0·966 | 0·955 | 0·943 | 0·932 | 0·919 | 0·908 | 0·895 | 0·882 |
| 5·5 | 0·426 | 0·988 | 0·979 | 0·966 | 0·956 | 0·944 | 0·932 | 0·920 | 0·909 | 0·896 | 0·883 |
| 6·0 | 0·408 | 0·988 | 0·979 | 0·967 | 0·956 | 0·944 | 0·932 | 0·920 | 0·909 | 0·896 | 0·884 |

Some uncertainty arises in the choice of the correct value of C. We have already had occasion to notice that the theoreti-

cal equations for the discharge are based on the assumption that the various quantities with suffix 2 relate to the section of minimum area and not to the area of the orifice itself. Now experiments have shown that the downstream pressure, measured at the plane of the orifice, is usually not much different from that at the vena contracta, whereas the area of the vena contracta is, as already remarked, appreciably less than that of the orifice. We should, strictly, in making the compressibility correction, take the value of $p_2$ and of $a_2$ at the vena contracta; but no great error is introduced if we take $p_2$ at the downstream plane of the orifice instead of at the vena contracta. On the other hand, some allowance for the reduction in area is necessary, since the true ratio of areas is not $\frac{a_1}{a_2}$ but $\frac{a_1}{\lambda a_2}$ or $\frac{r}{\lambda}$, where $\lambda$ is the ratio of the area of the vena contracta to the area of the orifice. Now $\lambda$ is not readily determinate by experiment, but an approximation to its value can be obtained by the following method.

The assumption is made that the area of the vena contracta is not affected by compressibility, i.e. that the shape of the stream on emerging from the orifice is the same as that for an incompressible fluid. It is unlikely that this assumption is strictly true, but if the value of $\frac{p_2}{p_1}$ is not too small it probably gives sufficient accuracy, since we are seeking a correction factor to C, which itself has the nature of a correction.

On this assumption, therefore, we may revert to the method by which the flow equation for an incompressible fluid was established, and derive an equivalent expression in a slightly different manner. We take $\lambda a_2$ as the area of the vena contracta, where $\lambda$ is less than unity, and, if we now take all conditions with suffix 2 as those obtaining at the vena contracta (except $p_2$, which, as we have seen, may be the value of the pressure at the downstream plane of the orifice), equation (4) becomes

$$q = \delta(\lambda a_2) g \sqrt{\frac{2\rho(p_1 - p_2)}{1 - \frac{\lambda^2 a_2^2}{a_1^2}}}, \qquad (14)$$

where $\delta$ is the ratio of the actual velocity through the vena contracta to the theoretical. This ratio will be less than unity on account of the frictional loss through the orifice.

The quantity $q$ as determined from equation (14) is that calculated from the conditions obtaining at the vena contracta, whereas in (5) it was determined from the conditions at the orifice. Equations (5) and (14) therefore represent the same quantity and hence we may equate their right-hand sides and obtain

$$\delta\lambda a_2\sqrt{\frac{a_1^{\,2}}{a_1^{\,2}-\lambda^2 a_2^{\,2}}} = \alpha a_2\sqrt{\frac{a_1^{\,2}}{a_1^{\,2}-a_2^{\,2}}},$$

or $$\frac{\delta^2\lambda^2}{r^2-\lambda^2} = \frac{\alpha^2}{r^2-1} \text{ (where } r = \frac{a_1}{a_2} \text{ as before)},$$

from which $$\lambda = \alpha r\sqrt{\frac{1}{\alpha^2+\delta^2(r^2-1)}}.$$

Hence $$\frac{r}{\lambda} = \sqrt{1+\frac{\delta^2}{\alpha^2}(r^2-1)} \quad . \quad . \quad . \quad (15)$$

Now the frictional loss through a sharp-edged orifice is known to be small. From experiments with water, the value of $\delta$ has been found to be between 0·96 and 0·998.* In making the approximation to $\frac{r}{\lambda}$ by equation (15) a mean value of 0·97 may therefore be taken for $\delta$, and if, further, we take $\alpha$ as 0·61, equation (15) reduces to

$$\frac{r}{\lambda} = \sqrt{2{\cdot}52r^2 - 1{\cdot}52}. \quad . \quad . \quad . \quad (16)$$

In calculating the value of C, therefore, the quantity $\frac{r}{\lambda}$, as given by (16), should be used instead of $r$ wherever the latter occurs in the formulæ.

It has already been pointed out that the variation of C with $r$ is not great when $r$ exceeds 3. Hence a rapid approximation to $\frac{r}{\lambda}$ may often be made by making use of the fact that the limit to which this ratio tends as $r$ becomes progressively larger is, as shown by equation (15), equal to $\frac{\delta r}{\alpha}$, or $1{\cdot}59r$ if $\delta = 0{\cdot}97$ and $\alpha = 0{\cdot}61$. The substitution $1{\cdot}59r$ may therefore be made for $\frac{r}{\lambda}$ in calculating C instead of the value given

* *Hydraulics*—Lea, 2nd Edition, p. 57.

by (16). The value of $\frac{r}{\lambda}$ calculated from this approximate expression when $r = 2$ is 3·18, and from equation (16), 2·92, and on reference to Table III. the difference in C is seen to be negligible. At higher values of $r$ the error is still less, but the approximation should not be used for values of $r$ less than about 1·7.

**Summary of Data for Sharp-Edged Orifices.**—We may now summarise the results for sharp-edged orifices of the type shown in Fig. 26 as follows :—

(1) For values of $\frac{d_2}{d_1}$ less than 0·7,* and for values of $\frac{p_2}{p_1}$ greater than 0·98, the value of the discharge coefficient may be taken as 0·61 without correction. If we exclude cases of the flow of compressed air, $\frac{p_2}{p_1}$ will not be less than 0·98 until the pressure drop across the orifice $(p_1 - p_2)$ exceeds about 8 inches of water.

(2) When $\frac{p_2}{p_1}$ is less than 0·98, $\alpha$ is given approximately by equation (13), and this value of $\alpha$ must be multiplied by a compressibility factor C, whose value is given by equation (9) and in Table III.

(3) In calculating C the ratio of areas $r$ must be replaced by $(1{\cdot}59r)$ provided that $\frac{d_2}{d_1}$ is less than 0·77 ($r = 1{\cdot}7$ approximately), otherwise the substitution $\sqrt{2{\cdot}52r^2 - 1{\cdot}52}$ must be employed.

The value 0·61 for the discharge coefficient only holds, within the limits noted above, for orifices of the type illustrated in Fig. 26, in which the upstream edges are sharp and truly square, and bevelled off on the downstream side, and when the pressures are determined at the walls of the pipe in the upstream and downstream planes of the orifice plate. It should also be remembered that the coefficient of discharge will almost certainly depend upon the velocity distribution in the pipe upstream of the orifice. We have already seen that in a given pipe the velocity distribution ultimately assumes a definite form, which is substantially the same for all pipes and does not vary much with rate of flow once the critical velocity is exceeded

* According to Jakob and Kretschmer this ratio may be as large as 0·85.

(see Fig. 18). It is found, however, that the air has to travel along a length of straight pipe measured from the inlet of at least 30 or 40 pipe diameters, before the characteristic distribution is finally assumed. At sections nearer to the inlet, disturbances due to the entry of the air into the pipe are liable to influence the distribution of velocity. This point was no doubt appreciated by Hodgson in his researches, and it is to be presumed that his coefficients relate to orifices placed in the region of what we may term normal distribution, that is to say in straight pipes, reasonably smooth, at the requisite distance from the inlet.

It appears, therefore, that orifices placed in regions where the velocity distribution is not normal may have different discharge coefficients, although they are otherwise similar in every respect to orifices having coefficients of 0·61, so that, in addition to the necessity for accurate workmanship in the making of the orifices, care has to be taken that they are installed in suitable lengths of pipe if they are to be used without calibration. For most practical purposes it will be sufficiently accurate to assume that Hodgson's coefficients will hold if the orifice plate is preceded by a straight length of pipe 10 diameters in length. In cases in which the necessary length of straight pipe is not available, or in which some doubt is felt as to the actual distribution of velocity, a preliminary calibration of the orifice should be undertaken. In such circumstances, it may prove advantageous to instal an orifice with slightly rounded edges, which is easier to construct and not so likely to suffer damage as the sharp-edged type, and to locate the pressure taps at any convenient points near to the plate, but not necessarily in the upstream and downstream planes.* A calibration can then be performed in order to determine the coefficient for that particular orifice, as installed. If the pipe is sufficiently large, the quantity of air flowing can be measured by a Pitot tube as already described ; otherwise one of the methods for small pipes (see Chapter VIII.) must be employed. At the same time the pressure difference across the orifice and the air temperature in the pipe and the barometric pressure at the time of

* It should be remembered, however, that the downstream pressure is a minimum at the vena contracta, so that the pressure difference to be measured will become progressively smaller as the downstream tap is moved further downstream from the vena contracta, until the position of maximum downstream pressure (minimum pressure difference) is reached where the pipe again runs full (see p. 87.)

measurement must also be observed. Equation (9) or (12) for the discharge through the orifice can then be solved for $\alpha$, by inserting the measured value of Q. If $\frac{p_2}{p_1}$ is less than 0·98, $p_1$ and $p_2$ must be measured in order to determine their ratio, and C must be calculated. Initially, before $\alpha$ is known, it will not be possible to determine the value of $\frac{r}{\lambda}$, according to equation (16), which must be used instead of $r$ in calculating C. It will then be necessary to calculate an approximate value of $\alpha$, using $r$ instead of $\frac{r}{\lambda}$ in evaluating C. From this approximate value of $\alpha$, a first approximation to $\frac{r}{\lambda}$ can be calculated from equation (16). C can then be re-calculated, using the approximate value of $\frac{r}{\lambda}$ thus determined, and a second approximation to $\alpha$ can then be found. If necessary this procedure can be repeated a stage further, but it will usually be found that the second approximation to $\alpha$ is sufficiently close to the real value.

**Orifice Plates in which the Pressure Connections are not at the Upstream and Downstream Planes.**—In their Report on Fluid Meters * the Special Research Committee of the American Society of Mechanical Engineers give numerous data compiled from the results of tests on sharp-edged orifices in which the pressure connections were at positions other than at the two planes of the plate. It would seem, however, that more reliance can be placed on Hodgson's experiments than on the others cited in the above-mentioned Report. It may be said, in fact, that Hodgson's results appear to provide the most trustworthy information on sharp-edged orifices at present in existence ; but there is still a wide field to be explored before the characteristics of these devices can be regarded as completely elucidated.

**Loss in Pressure Due to the Passage of Air through an Orifice Plate.**—As we have seen, the static pressure at the downstream face of an orifice plate is considerably less than that in the air upstream of the plate. There is a further slight decrease as the air leaves the orifice, a pressure minimum being reached at the vena contracta at about ½ to 1 pipe diameter

* Published by the American Society of Mechanical Engineers, New York.

from the orifice. This pressure minimum is followed by a gradual recovery of pressure which persists up to a distance of about three to ten or more diameters downstream of the orifice, the actual distance increasing with the ratio $r$ and probably depending also on the rate of flow. At some point, therefore, a pressure maximum is experienced, which corresponds to the position where the jet, which at the vena contracta had a section considerably less than that of the pipe, has expanded once more to the full pipe diameter. After this pressure maximum has been reached the flow proceeds to follow the normal laws of pipe flow, and the pressure falls off gradually on account of the ordinary frictional loss at the walls. The point to be observed is that the static pressure at the down-

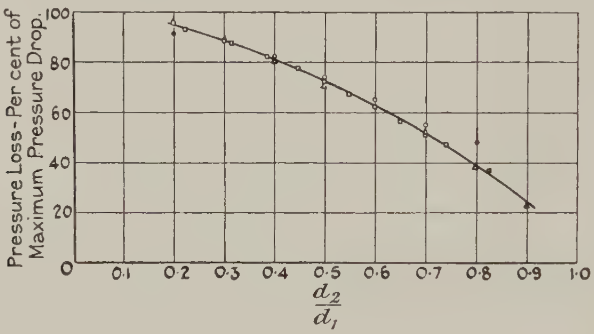



FIG. 27.—Pressure loss due to orifice plate.

stream point of maximum recovery is considerably less than that upstream of the orifice, and also considerably less than the static pressure that would be experienced at that point if the orifice were removed and the normal frictional pressure drop in a parallel pipe existed between the upstream position and the downstream position corresponding to the point of maximum recovery with the orifice in place. The orifice, in fact, introduces a resistance equal to that of a long length of pipe, and causes a corresponding loss of head or energy in the flow. In other words, a fan delivering a given quantity of air per unit time along a pipe line would take more power if an orifice were introduced into the line and the quantity of air flowing were maintained at its previous value.

Some idea of the pressure loss experienced by air in its passage through an orifice may be gained from Fig. 27 which is taken from the Research Report on Fluid Meters of the

American Society of Mechanical Engineers, and represents the mean of the results obtained by a number of workers. The static pressure drop between an upstream point near the orifice and the downstream point of maximum recovery is expressed as a percentage of the pressure difference between the same upstream point and the vena contracta (i.e. the maximum pressure difference across the orifice), and is plotted on a base of $\frac{d_2}{d_1}$. It may be noted that this curve agrees fairly well with similar data given by Hodgson,* when allowance is made for the fact that Hodgson measured his downstream pressure in the plane of the orifice.

**The Venturi Tube.**—The factor which contributes mostly to the loss of head through a plate orifice is the sudden increase of area after the air has passed the orifice ; the rapid convergence of the stream on the upstream side as it contracts to pass through the orifice is found to be of minor importance. The cause of the loss of head on the downstream side seems to be that the space between the boundaries of the jet and the walls of the pipe before the jet has expanded again to the full pipe area, becomes filled with " dead air," in a state of violent turbulence, in which an appreciable amount of energy is dissipated. If the transition from the full pipe diameter to the minimum section and the subsequent re-expansion to the full area are carried out gradually, by actually guiding the air through conical lengths of pipe of fairly small taper (see below), the pressure drop from the upstream entry to the minimum section, or " throat," may be in large measure recovered after expansion. An arrangement of this kind, in which the section of the pipe is gradually reduced and subsequently expanded to its original area, is called a Venturi tube, and it may, in certain cases where it is desired to avoid loss of head, be preferred to an ordinary orifice plate. It has the disadvantages that it is more expensive, and that its insertion in an existing pipe line necessitates more alterations and fitting than that of an orifice plate.

The Venturi tube is usually constructed of approximately the proportions shown in Fig. 28. The apex angle of the converging upstream cone is about 20°, and that of the diverging downstream cone about $5\frac{1}{2}°$, changes of slope being made through easy curves. In order to obtain good mean values of the static pressure at the entrance and the throat, these

* *Proc. Inst. Civil Engineers*, loc. cit., Fig. 18.

sections are usually surrounded, as in the diagram, by annular chambers, or "piezometer rings," connection being made to the inside of the tube by means of a series of small holes pierced round its circumference. Two tubes, one from each annular chamber, are led to a suitable manometer.

The slopes of the inlet and outlet cones have been fixed by experience. In order to reduce the wall frictional losses through the Venturi to a minimum, it is obviously desirable to make the instrument as short as possible, i.e. to make the divergence and convergence as rapid as possible. As regards the exit cone, however, it is found that a diverging conical pipe having a total included angle of more than about 7° does not "run full," that is to say, there is a tendency for the boundaries of the stream to leave the walls and give rise to a "dead water" region which, as in the case of the jet from an orifice, leads to a loss of head. For this reason the exit cone must be long and gently sloping, the maximum permissible angle being about 7°. The

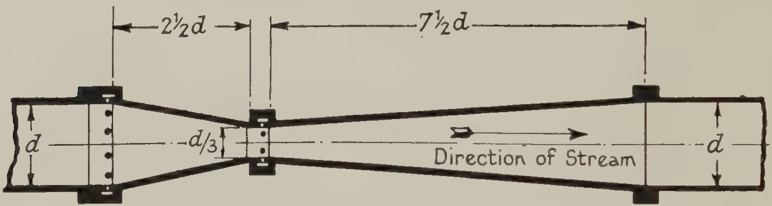



FIG. 28.—Venturi tube.

unavoidable increase in the wall friction consequent upon the long exit cone is small compared with the loss of head experienced if the exit cone does not run full. The inlet cone, in which there is no tendency for the stream to leave the walls, can be made considerably shorter with its walls sloping at a steeper angle. A limit on the slope is imposed in this case by the consideration that the throat of the Venturi must run full and that the direction of streaming there must be everywhere parallel to the axis of the pipe. If the slope of entry is too steep there is a tendency for a vena contracta to be formed in or near the throat. It is probable that an entry of a form similar to that of a shaped nozzle (see Fig. 30) could be used with advantage and be considerably shorter than the conical inlets in common use on Venturis; but it would, in view of the more careful shaping necessary, be more expensive to construct than a simple converging cone. In practice, a 20° conical inlet is found to fulfil requirements and has been generally adopted.

The quantity of air flowing per minute through a Venturi is calculated by the use of equation (12), the quantity $r$ now being taken as the ratio of the area at the commencement of the entry cone to the area of the throat, whilst $h_1$ and $h_2$ are the pressures at these sections. As in the case of the orifice, if $\frac{p_2}{p_1}$ is less than about 0·98 the value of the compressibility factor C in equation (12) begins to differ appreciably from unity, and must be determined. Since the minimum area of the jet in a properly designed Venturi is equal to the area of the throat, there is no need to apply a correction, similar to that necessary for a plate orifice, to the value of $r$ in calculating C.

The coefficient of discharge $\alpha$ of a Venturi is, on account of the absence of any contraction of the stream after it issues from the throat, considerably greater than the normal discharge coefficient of a plate orifice, being, in fact, usually between 0·95 and 0·98. There does not, unfortunately, appear to be sufficient published information to enable the coefficient to be stated with a great degree of precision. In the first place, no particular design of Venturi has been adopted as a standard, and, as was pointed out in discussing orifice discharge, the coefficient must to some extent depend upon the shape and proportions of the constriction. Secondly, most of the published data obtained from systematic tests relate to the flow of water. A useful analysis of these data has been made by the American Society of Mechanical Engineers in their Report on Fluid Meters (Part I.), where the information is presented in such a form that it may be applied, within certain limits, to the flow of air.

We have already seen (equation (11)) that the coefficient $\alpha$ in the case of the flow of an incompressible fluid through any one of a series of geometrically similar constrictions is some function of the quantity $\frac{v_2 d_2}{\nu}$, where $v_2$ and $d_2$ are the velocity at the throat and the diameter of the throat respectively, and $\nu$ is the kinematic viscosity of the fluid. All the quantities must be in terms of a consistent set of units—in England the foot-pound-second system is usually adopted. It has also been shown that the use of this criterion is not restricted to any particular fluid. For instance, two constrictions geometrically similar to each other will have the same values of $\alpha$ when one is delivering air and the other water, provided that $\frac{v_2 d_2}{\nu}_{\text{air}}$ for

the one is equal to $\frac{v_2 d_2}{\nu_{\text{water}}}$ for the other, and provided also that the pressure changes in the air are small enough to warrant its being regarded as incompressible. We can thus make use of a water calibration to predict with confidence the discharge coefficient for air, since at the same value of $\frac{v_2 d_2}{\nu}$ the coefficients will be the same. This statement, which is a particular instance of the general law of " dynamical similarity " as applied to fluid motion, has a rational theoretical basis and has been fully substantiated by experiment.

Making use of this law, the American Society of Mechanical Engineers * have plotted on a base of $\frac{v_2 d_2}{\nu}$ reliable data, obtained from various experimental sources, relating to Venturi coefficients, and have found that a mean curve can be drawn through all the experimental points, none of which shows a departure exceeding $2\frac{1}{2}$ per cent. of the mean value of $\alpha$ given by the curve. Included in the data are figures for Venturi tubes having $\left(\frac{\text{maximum diameter}}{\text{throat diameter}}\right)$ ratios ranging between 2 and 3. Strictly, such a curve could only be expected to apply to geometrically similar Venturi tubes of constant ratio, but the fact that such good consistency is actually found indicates that a departure from similarity between the above limits does not seriously affect the value of $\alpha$. The curve is reproduced in Fig. 29 on an actual base of $\log \frac{v_2 d_2}{\nu}$ instead of $\frac{v_2 d_2}{\nu}$. This method of plotting is adopted in order to include all the data in a diagram of reasonable size ; plain plotting on a $\frac{v_2 d_2}{\nu}$ base would necessitate a very long figure or, alternatively, one in which the region relating to the higher rates of flow would be unduly compressed. The American Society of Mechanical Engineers consider that values of $\alpha$ taken from this curve may be regarded as accurate within 1 per cent. for Venturi tubes of normal commercial design and construction under normal conditions of installation and operation, to which proviso must be added, in the case of air flow, the condition that the ratio $\frac{p_2}{p_1}$ should not be less than 0·98.

* *Loc. cit.*

A variation of $\alpha$ with $\frac{p_2}{p_1}$, of a nature related to the similar variation observed with plate orifices, has been recorded by Gibson * in tests on a small Venturi having a throat diameter

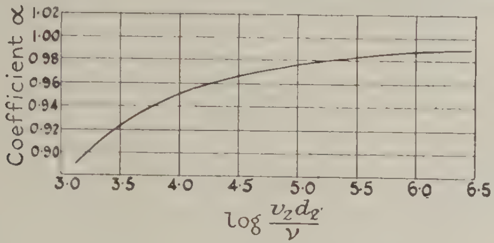



FIG. 29.—Venturi coefficients.

of $1\frac{1}{2}$ inches in a 3-inch pipe. He found, as shown in Table IV., that $\alpha$ increased by about 8 per cent. as $\frac{p_2}{p_1}$ was diminished from 1·0 to 0·5.

TABLE IV.

| $\frac{p_2}{p_1}$ | 0·5. | 0·6. | 0·7. | 0·8. | 0·9. | 1·0. |
|---|---|---|---|---|---|---|
| $\alpha$ | 0·98 | 0·98 | 0·97 | 0·96 | 0·94 | 0·91 |

It is of interest to compare Gibson's results with the curve of Fig. 29. The limiting value of $\alpha$ shown in the above table as $\frac{p_2}{p_1}$ approached unity was obtained by extrapolation of the experimental curve and does not, of course, represent an actual condition of flow, which must be zero when $\frac{p_2}{p_1} = 1$. We may therefore suppose without much error that Gibson's Venturi had a coefficient of about 0·92 (probably somewhat less) at a value of $\frac{p_2}{p_1}$ of 0·99. Now for this value of $\frac{p_2}{p_1}$, which, if $p_1$ be taken as normal atmospheric pressure, corresponds to a pressure difference $p_2 - p_1$ of about 4 inches of water (say 21 pounds per square foot), equation (7*a*) shows that the velocity at the

* *Proc. Inst. Mech. Engineers*, October-December, 1919, p. 593.

throat of this particular Venturi was about 138 feet per second. Taking $\nu$ as 0·00016 we therefore find, since $d_2 = 1{\cdot}5$ inches $= \frac{1}{8}$ foot, the value of $\frac{v_2 d_2}{\nu}$ to be 108,000 approximately, so that $\log \frac{v_2 d_2}{\nu} = 5{\cdot}03$ and the value of $\alpha$ from Fig. 29 should be about 0·976. There is thus a discrepancy between Gibson's figures and those of Fig. 29. It should be noted, however, that the exit cone of the Venturi used by Gibson had a steeper slope than that commonly used. The expansion from $1\frac{1}{2}$ inches diameter at the throat to the full 3 inches diameter occurred in a length of $6\frac{1}{2}$ inches, which corresponds to an apex angle of the cone of 13°. This was almost certainly too large an angle for the cone to run full, particularly at the high throat speeds used in these experiments, and it is possible that this factor may have contributed to the relatively small value of $\alpha$ at the rates of flow at which the air could legitimately be regarded as incompressible. In the absence of further information no definite conclusion can be drawn, but the remarkable agreement obtained between different sets of experimental results in the derivation of the American curve of Fig. 29, and the fact that this method of correlating data rests on a secure theoretical basis, would seem to justify the practical application of this curve, within the limits stated, to derive values of $\alpha$ in all cases of air flow where the highest accuracy is not required. In special cases a calibration of the Venturi in situ may be performed in the manner previously described for an orifice calibration. Gibson's results may be used to give an indication of the variation that may be expected in $\alpha$ with changes in the expansion ratio.

In order to deduce the discharge coefficient from Fig. 29 a knowledge is required of $v_2$, the throat speed, and $\nu$. The former can be determined from equation (3*a*), treating the air as incompressible. This is only permissible if $\frac{p_2}{p_1}$ is not much different from unity, but it must be remembered that Fig. 29 only holds for incompressible flow, since it was derived from experiments with water, so that it cannot be relied upon to give good accuracy if $\frac{p_2}{p_1}$ is less than, say, 0·95. Values of $\nu$ for air in foot-pound-second units are given for a range of temperature in the following table :—

TABLE V.

*Values of $\nu$ for Air.*

| Temperature. Deg. Cent. | Value of $\nu$. | Temperature Deg. Cent. | Value of $\nu$. |
|---|---|---|---|
| 0 | $144 \times 10^{-6}$ | 40 | $185 \times 10^{-6}$ |
| 5 | $149 \times 10^{-6}$ | 50 | $196 \times 10^{-6}$ |
| 10 | $154 \times 10^{-6}$ | 60 | $207 \times 10^{-6}$ |
| 15 | $159 \times 10^{-6}$ | 70 | $218 \times 10^{-6}$ |
| 20 | $164 \times 10^{-6}$ | 80 | $229 \times 10^{-6}$ |
| 25 | $169 \times 10^{-6}$ | 90 | $240 \times 10^{-6}$ |
| 30 | $174 \times 10^{-6}$ | 100 | $251 \times 10^{-6}$ |

The pressure loss in a Venturi is appreciably less than that caused by an orifice, about 80 or 90 per cent. of the drop from entry to throat being ultimately recovered. (Compare Fig. 27.) Otherwise, the Venturi in its practical aspects presents much the same features as the orifice, and the remarks made under the latter heading regarding the relatively high pressures obtainable at low speeds, and the facility with which continuous records of flow can be taken, apply equally to the Venturi. As with the orifice plate, it is also important to have a sufficient length of straight pipe upstream of the entry.

**The Shaped Nozzle.**—The chief disadvantages of the plate orifice are the uncertainty that exists regarding the area of the vena contracta and the relatively large resistance it offers to the passage of air through it. Both these undesirable features are, as we have seen, avoided by the use of a Venturi, but only at the expense of considerably increased cost and installation difficulties. The shaped nozzle, which is, in effect, a Venturi with the expansion cone removed, is a device intermediate between the two types of constriction hitherto considered. The unrecoverable loss of pressure through it is only slightly less than that through a plate orifice, and in this respect it is inferior to the Venturi. It possesses, however, the important feature of the latter that the jet at the section of low-pressure measurement is parallel and of an area equal to that of the minimum area of the constriction itself, since no subsequent contraction occurs. As regards cost and difficulty of installation it is superior to the Venturi, and only slightly worse than the plate orifice. Unless ease of installation and first cost are prime considerations, it is the author's opinion that in the

majority of cases the shaped nozzle is the best form of constriction to use.

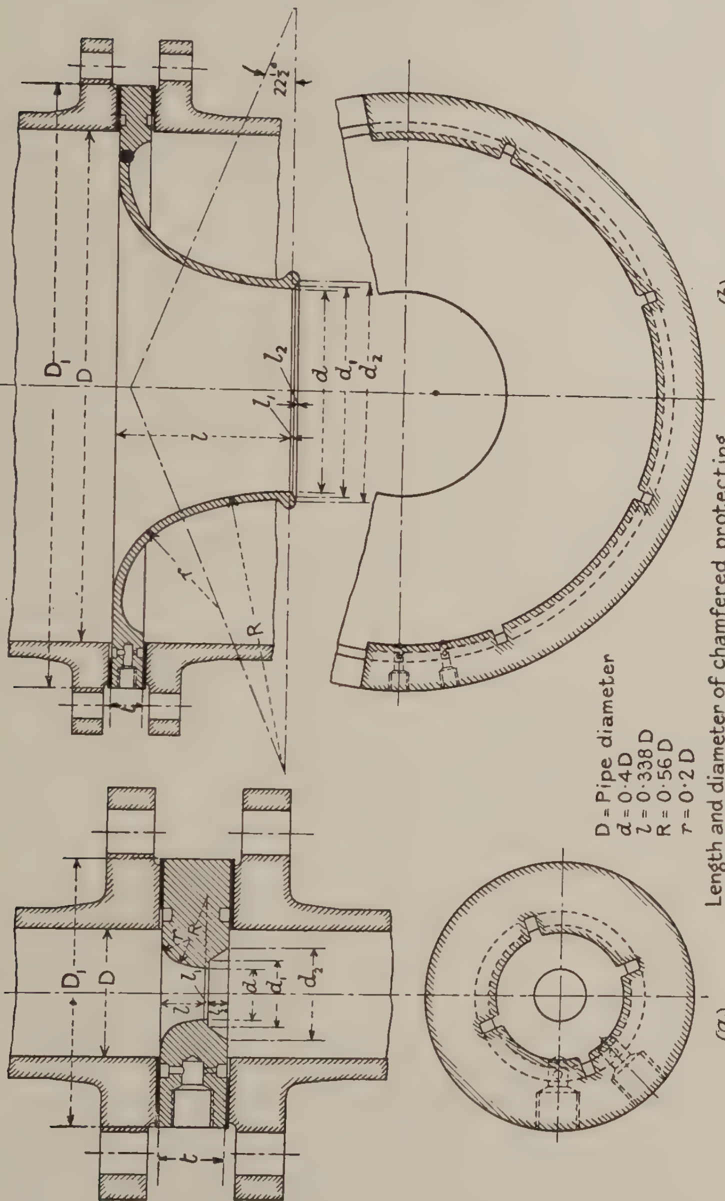
Two types of shaped nozzles are illustrated in Fig. 30. It will be seen that there is a curved entrance terminating tangentially in a direction parallel to the axis of the pipe. Some types have a short cylindrical throat at the end of the curved entrance, but this is found not to be essential, as, by properly shaping the curve, the fluid can be made to issue from the nozzle in a straight cylindrical jet without any parallel walls to guide it. The pressure drop through the nozzle is measured near the entry and at the throat, as shown in the diagram, and the flow is calculated, as for a Venturi, by the use of equation (12). The discharge coefficient for a well-designed nozzle is about 0·95, varying somewhat with the design.

The nozzles illustrated in Fig. 30 have been standardised by the Association of German Engineers in their rules for the testing of fans and air-compressors. The thick plate type is used for small pipes up to 70 mm. (2·76 inches) in diameter, where weight is not objectionable, and the other (Fig. 30*b*) for larger pipes. The principal relative proportions are shown in the diagrams, and the leading dimensions for a number of pipe sizes in Table VI.

TABLE VI.

*German Standard Nozzles. Dimensions in Millimetres.*

| Pipe Diameter. | Throat Diameter. | R. | *r*. | Length of Nozzle Proper. | Length of Cylindrical Portion. | Length and Diameters of Chamfer. | | | Nozzle Flange Thickness. |
|---|---|---|---|---|---|---|---|---|---|
| D. | *d*. | | | *l*. | $l_1$. | $d_1$. | $l_2$. | $d_2$. | *t*. |
| 25 | 10 | 14 | 5 | 9 | 1 | 14 | 16 | 22 | 26 |
| 50 | 20 | 28 | 10 | 17 | 1 | 26 | 8 | 36 | 26 |
| 70 | 28 | 39 | 14 | 24 | 1 | 34 | 3 | 38 | 28 |
| 85 | 34 | 48 | 17 | 29 | 1 | 40 | 3 | 44 | 28 |
| 100 | 40 | 56 | 20 | 34 | 1 | 46 | 3 | 50 | 28 |
| 150 | 60 | 84 | 30 | 51 | 1 | 68 | 3 | 72 | 28 |
| 200 | 80 | 112 | 40 | 68 | 1 | 88 | 3 | 92 | 29 |
| 500 | 200 | 280 | 100 | 169 | 2 | 210 | 4 | 216 | 30 |
| 1000 | 400 | 560 | 200 | 338 | 3 | 410 | 5 | 416 | 30 |
| 1500 | 600 | 840 | 300 | 507 | 4 | 610 | 6 | 618 | 30 |
| 2000 | 800 | 1120 | 400 | 676 | 5 | 810 | 7 | 820 | 30 |
| 2500 | 1000 | 1400 | 500 | 844 | 6 | 1010 | 8 | 1020 | 30 |




$D$ = Pipe diameter
$d = 0{\cdot}4D$
$l = 0{\cdot}338D$
$R = 0{\cdot}56D$
$r = 0{\cdot}2D$

Length and diameter of chamfered protecting portion on throat varies with size of nozzle.

FIG. 30.—German standard nozzle.

Pressures are measured at the upstream and downstream faces of the flange of the nozzle, by means of pressure taps leading, as shown in Fig. 30, to annular grooves cut in the two faces of the flange. Between each pipe flange and the adjacent face of the nozzle flange is inserted a packing ring which covers and seals the annular pressure groove entirely, except at a number (4 to 20, according to the size of the nozzle) of places around the circumference, where slots cut in the ring put it in direct communication with the pipe static pressure at these points and so enable the mean pressure to be led to the manometer through the pressure tap connected to the ring.

The determination of the discharge coefficient of these nozzles formed the subject of a careful investigation by Jakob and Erk * who found that the coefficient was sensibly constant for values of $\frac{vd}{\nu}$ at the throat ranging from 71,000 to 377,000. The maximum drop across the nozzle in these experiments was $6\frac{1}{4}$ inches of water, so that equation (5) for incompressible flow could be used, the absolute pressure in the pipes being always approximately atmospheric. Since in this standard nozzle the ratio of the pipe diameter to the throat diameter is specified as constant for all sizes and equal to 2·5, it saves numerical work to obtain a value of the expression $\alpha\sqrt{\frac{r^2}{r^2 - 1}}$ instead of $\alpha$ only, and to call this the coefficient of the orifice. Expressing their results in this manner, Jakob and Erk found the coefficient within the above-mentioned range of $\frac{vd}{\nu}$ to have the mean value 0·96, whence, since $r$, the ratio of the areas, is 6·25, $\alpha$ for this nozzle becomes 0·948. Earlier experiments on this standard nozzle, in which the quantity of air flowing was measured by means of a calibrated gasometer, had indicated a coefficient of 0·97 to 0·995, but the various practical difficulties attendant upon this method of measurement (as, for example, the determination of the true mean temperature of the air in a large container such as that used, which had a volume of about 175,000 cubic feet) rendered the results not entirely satisfactory, and in presenting them it was stated that further tests were desirable. In Jakob and Erk's experiments the

* *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, No. 267, 1924, *Der Druckabfall in Glatten Rohren und die Durchflussziffer von Normaldüsen*, published by the Verein Deutscher Ingenieure.

quantity was determined by measuring the pressure drop in suitable lengths of smooth calibrated pipes, a method which, as we have seen,* is capable of giving very good accuracy when the proper experimental conditions are observed.

An important feature of these experiments is that they included tests of the nozzles both with a length of pipe on the outlet side, and when placed at the end of the pipe line and discharging into atmosphere. Although the tests under each of these two conditions were sufficiently numerous to detect any difference in the discharge coefficient that might have existed, no such effect was found, and it may be accepted that the coefficient is the same in both cases. This result is of some practical importance, since it provides a ready means of measuring the quantity of air flowing in a small pipe (see p. 139).

A small number of tests was also made in which the nozzle was placed in a pipe of considerably greater diameter than the standard size for the nozzle. The discharge coefficient (as defined in these tests, viz. $\alpha\sqrt{\frac{r^2}{r^2 - 1}}$) was found to be 0·972. Now if $\alpha$ is the same for this type of nozzle irrespective of the size of the pipe in which it is installed, the quantity $\alpha\sqrt{\frac{r^2}{r^2 - 1}}$ will decrease as $r$ increases, and will be equal to $\alpha$, i.e. to 0·948, when $r$ becomes very large—a condition corresponding to that of air discharging to atmosphere from a large chamber through a nozzle placed in the side. In a pipe larger than the standard size, therefore, Jakob and Erk should have found the value of their coefficient to be less than 0·96. In the conclusions to their papers, however, these experimenters do not seem to have attached much importance to this apparent discrepancy; they state that the value 0·96 for $\alpha\sqrt{\frac{r^2}{r^2 - 1}}$ may be adopted for these nozzles irrespective of conditions of installation, and regard the higher value obtained in the three tests made in larger pipes in the nature of an unexplained experimental error. The position does not, however, appear to be quite as satisfactory as this statement implies. There is no *a priori* reason why the discharge coefficient should be constant except for similar nozzles similarly installed, and whilst their conclusion as to the general applicability of the value 0·96 is probably sufficiently

* Compare Stanton and Pannell's experiments, p. 39.

accurate for most purposes where the highest precision is not required, this value can only be regarded as definitely established for the case in which the throat diameter is equal to 0·4 times the pipe diameter. For this condition the coefficient may be accepted without hesitation, provided, as with the Venturi tube and the plate orifice, a sufficient length of straight pipe—say 10 diameters at least—precedes the entry to the nozzle. Further research on the value of the coefficient for ratios of throat to pipe diameters differing considerably from 0·4 would be of value, particularly if the tests were extended to include the limiting case of the discharge from large chambers, which is a measurement that frequently occurs in practice, and for which the use of a calibrated nozzle is the most convenient method.

One further characteristic of these nozzles, which, as we shall see later, is of some practical importance, deserves mention. Careful explorations of the issuing jet in a number of instances disclosed the fact that the velocity was practically constant over the entire section. It was only in the immediate vicinity of the walls that a slight falling off was observed. For all practical purposes the jet may be considered as having a uniform velocity equal to that measured by a single observation at the axis.

# CHAPTER VII.

## THE VANE ANEMOMETER.

ALTHOUGH it is subject to certain practical limitations, which will be indicated later, the vane anemometer has been extensively employed for many years. As a means of measuring wind speed, particularly in the range of low speeds, where, as we have already seen, such small pressures are set up by the motion that their accurate determination becomes a matter of considerable difficulty, its simplicity renders it an exceedingly useful instrument; and if properly used it is capable of giving all the precision ordinarily required in engineering practice.

The vane anemometer is, in effect, simply a windmill, consisting of a number of light flat vanes mounted on radial arms attached to a common small steel spindle which rotates in two jewelled bearings. Eight vanes, made of thin sheet aluminium or aluminium alloy, are almost invariably used; they are inclined at an angle to the axis of the spindle, which is set along the wind direction when the instrument is in use, and the wind forces acting on the vanes cause the spindle to rotate at a rate depending mainly on the air speed.* By means of suitable gearing the motion of the spindle is communicated to a pointer, or, more usually, to a number of pointers moving over graduated dials. Normally, the dials are marked with scales of feet, and experience has taught manufacturers of these instruments how to proportion the various dimensions, vane inclinations, and gearing ratios, so that the number of feet indicated on the dials in unit time is approximately equal to the distance traversed by the wind in the same time, i.e. to the wind speed. In order, therefore, to determine the wind speed it is only necessary to observe with the aid of a stop watch the time taken for a number of feet of air, as shown by the indicating mechanism, to pass the instrument. This gives the "indicated" wind speed, from which the true air speed is

* See below for the effect of air density.

obtained by the aid of the calibration curve for the particular anemometer in use.

Instruments of this type are mainly of value for measuring air speeds in large ducts or ventilating shafts, or in smaller pipes (subject to the restriction as to the limiting size mentioned below) where access can conveniently be made to an open end, preferably an outlet end. They are probably best used when mounted on light rods and inserted into the airstream from one side. It is important that the observer should not stand too close to the instrument whilst readings are being obtained, as his presence will disturb the conditions of flow even if he is


FIG. 31.—Vane anemometer.

situated downstream of the position occupied by the instrument. In order to determine the quantity of air flowing, readings must be taken with the axis of the vane circle at a number of points across a section (see page 108).

Two types of anemometers in common use are illustrated in Figs. 31 and 32. In the former the plane of the dial is perpendicular to the plane of rotation of the vanes; whilst this design is not so compact as the other, it is probably to be preferred on account of the greater facility which it offers for taking observations from the side.

In many instruments provision is made for throwing the indicating mechanism into gear or disconnecting it from the vane spindle by the movement of a small lever. In some

instruments, also, the reading can be set to zero at any stage. These devices would appear to be designed to obviate the use of a stop watch, readings presumably being taken by throwing the indicating mechanism into gear at a given instant and disconnecting it when a certain time has elapsed. This method of using vane anemometers is not to be recommended; more accurate results will certainly be obtained by observing with


FIG. 32.—Vane anemometer with concentric dial.

a stop watch the time taken for a given number of complete revolutions of one of the pointers. The other method should only be adopted if a stop watch is not available. In order to reduce starting and stopping errors to small percentages of the total readings, no observation should occupy less than 60 seconds, or involve a reading of less than 100 feet on the instrument. It is advisable also to take two check readings of every observation, as it will often be found that the flow is

subject to fluctuations, so that the average over a given period is not quite constant.

It is often stated that vane anemometers are unreliable and that their indications are subject to unaccountable inconsistencies. The author's experience with these instruments does not support this contention. Trouble frequently arises through the use of an anemometer to measure wind speeds for which it is not designed. If the speed is too high, the vane setting may suffer a permanent distortion which will cause a corresponding permanent alteration in the calibration curve. On the other hand, if the speed is too low the friction of the bearings and gearing exercises an appreciable effect (see below). The normal types of instrument, such as those illustrated in Figs. 31 and 32, are suitable for the measurement of air speeds ranging from about 5 to 50 feet per second, and may confidently be relied upon over this range. For higher speeds, special instruments are made which are either of heavier construction, or, alternatively, incorporate a device by means of which the quantity of air actually passing through the vanes can be reduced ; such instruments can be used for wind speeds up to 100 feet per second or more.

**Low-Speed Anemometers.**—For low wind speeds it is essential to have an instrument in which the friction is very small. In order that the wind torque causing rotation should be large compared with the resisting frictional torque it is desirable that the vanes should be large and set at a large radius from the axis. These dimensions are, however, limited by considerations of lightness, for if the inertia of the moving parts is too great the instrument will not follow rapid fluctuations in the speed of the wind and will give readings which differ perceptibly from the mean wind speed. Moreover, it must be remembered that the resisting frictional torque is composed partly of a torque due to the weight of the moving parts, and partly of a torque due to the end thrust of the spindle on its downstream bearing, arising from the down-wind component of the wind force on the vanes ; the first of these factors will increase with the weight of the vanes and the second with their size. As is usual in design generally, a compromise has therefore to be effected between conflicting requirements.

A form of low-speed anemometer which gives consistent readings down to wind speeds of about 0·6 feet per second was designed by the author and described in the *Journal of Scientific Instruments* for January, 1926. This instrument is illustrated

in Fig. 33. The vanes are of the usual pattern, inclined at an angle of 45° to the axis, the outer radius of the vane circle is $1\frac{7}{8}$ inches, and the vanes are $1\frac{1}{8}$ inches deep. All gearing is eliminated with the exception of a worm drive through which the rotation of the vanes is transmitted, with a reduction of 50 to 1, to a pointer moving over a dial attached to one side of the casing. The pointer is fixed directly to the worm-wheel spindle, and the only markings on the dial are four short radial lines, as shown, at the ends of two mutually perpendicular

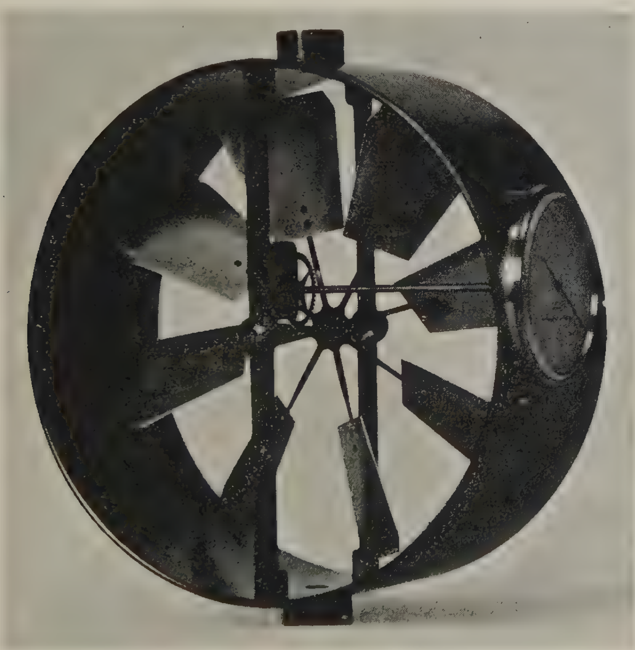


FIG. 33.—Author's low-speed vane anemometer.

diameters. The calibration of the instrument is given in terms of the relation between the wind speed and the revolutions per second of the pointer over the dial. Three tapped bosses are fitted to the casing (the third, which cannot be seen in the photograph, is opposite to the dial) for convenience in mounting the instrument to suit various conditions of use.

It is important to observe that every anemometer should be calibrated in a wind whose speed can be controlled and measured by independent means, or by moving it at known

speeds through still air, for it is not possible to predict the readings of an instrument from its mechanical constants. Until recently, instruments were frequently sent out by the makers accompanied by a printed form stating that a certain number of feet per minute must be added to the readings to give the true wind speed, the number being the same for all speeds. Such instructions should be regarded with suspicion and not accepted without verification. During the course of a large number of calibrations of vane anemometers carried out under his supervision, the author does not recollect a single instrument whose error was constant over the whole range.

The method of carrying out an anemometer calibration is in itself a simple matter, but considerable care must be taken to obtain an air stream suitable for the purpose. In cases where the amount of work necessitating the use of an anemometer does not warrant the provision of special testing equipment, the instrument may be submitted to the National Physical Laboratory for calibration.

**Readings in an Inclined Wind.**—The readings of a vane anemometer do not appear to be seriously affected by fairly large variations of the direction of the wind from its normal path perpendicular to the vane circle. Whilst it is not possible, on account of the numerous differences in details of shape and construction that occur amongst existing types of vane anemometers, to draw generalised conclusions from the observed behaviour of a particular instrument, Fig. 34 will serve to show what order of error may be expected if the wind is inclined to the axis of the vane circle. The results relate to experiments carried out by Messrs. F. C. Johansen and W. J. Duncan with a calibrated anemometer of more or less normal proportions. The instrument was mounted in a wind tunnel side by side with a second calibrated instrument which served as a standard. A sufficient distance was allowed between the two instruments, which were situated in the same cross-section of the tunnel, to render negligible any interference one might have on the other. Simultaneous readings of the two instruments were taken, the standard being kept stationary in normal attitude and indicating the true mean speed over the intervals of observation, whilst the other was rotated successively to the wind direction, by stages of 5° or 10° at a time, through large angles on each side of its normal attitude. At each setting the indicated speed of each instrument was observed, and the

ratio $\frac{V_\theta}{V_s}$ was calculated, $V_\theta$ being the speed as given by the yawed instrument at an inclination $\theta°$ and $V_s$ the true wind speed as indicated by the standard.

It will be seen from Fig. 34, where $\frac{V_\theta}{V_s}$ is plotted against

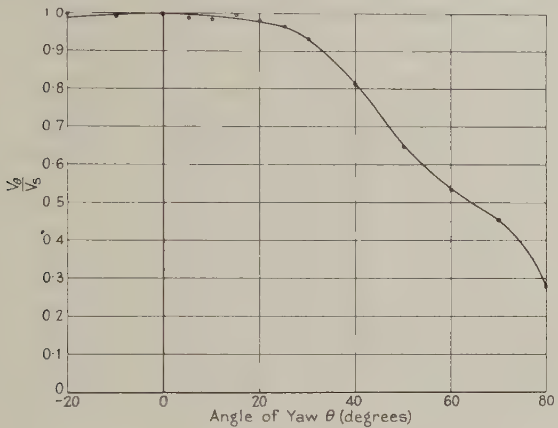



FIG. 34.—Effect of yaw on readings of vane anemometer.

$\theta$, that the instrument could be yawed through about 20° before an error of 1 per cent. on the indicated speed was incurred. It would appear therefore that little apprehension need be felt as to the accuracy of the readings of a vane anemometer if the mean direction of the air current is not known within fairly wide limits.

**Characteristics Indicated by Theory.**—The theory of the vane anemometer will be considered in the second part of the present chapter, but since the theoretical treatment is unavoidably somewhat involved, it will be useful at this stage to state the main deductions to which it leads. Under normal conditions an anemometer is generally used to measure wind speeds appreciably higher than the lowest speed which will just set the vanes in rotation. In these circumstances it is found that the effect of friction at the bearings can be neglected, and it will be shown that the rotational speed of the vanes will then depend only on the velocity of the air and not on its density. The readings of a well-made anemometer, in which the frictional resistance is very small, do not therefore require correction for atmospheric pressure and temperature provided that the vanes

are not rotating too slowly. For special low-speed work variations of the density of the air may, however, become important, and should be taken into account by the method shortly to be described.

As we have already seen in dealing with the flow of air in pipes, there is in general a variation in velocity from point to point across a section of a pipe, from which it follows that when an anemometer is used to measure the speed of air flowing along a pipe there will usually be a variation of wind speed across the anemometer disc. Theory shows that under these conditions the anemometer does not indicate accurately the speed of the filament of air passing through the centre of the disc, and that quite important errors may arise from this cause. It appears therefore that, in order to avoid such errors, the diameter of the anemometer should be fairly small in comparison with that of the pipe, so that the variation of speed across the instrument itself is small. Unless precautions are taken to equalise the velocity at different points along the pipe diameter, if a marked variation originally exists, an anemometer should not be used in a pipe whose diameter is less than about six times, preferably eight times, that of the instrument.

**Measurement of the Rate of Flow in Pipes with Vane Anemometers.**—Like the Pitot tube, the vane anemometer measures a local velocity which, subject to the limitations mentioned above, may be taken as the velocity of the filament of air passing through the centre of the vane wheel. In order, therefore, to determine the quantity of air flowing along a pipe by the use of this instrument, readings of velocity must be taken at a number of points across the section whose positions are determined in the manner indicated in Chapter V.; in other words, readings must be taken with the axis of the vane wheel at the points at which the velocity head would be observed if a Pitot tube were being employed for the purpose. If $v_1$, $v_2$, $v_3$, etc., are the air speeds in feet per second thus observed, the mean speed is given by

$$v_m = \frac{1}{n}(v_1 + v_2 + v_3 + \ldots),$$

where $n$ is the number of readings taken.

The volume of air flowing in cubic feet per minute is

$$V = 60Av_m,$$

where A is the area of the pipe section in square feet, and, since 1 cubic foot of dry air at 60° F. and 29·92 inches of mercury barometric pressure weighs 0·0763 pound, the weight of air flowing in pounds per minute is

$$Q = 60Av_m \times 0{\cdot}0763 \times \frac{b}{29{\cdot}92} \times \frac{520}{460 + t}$$

$$= 79{\cdot}6Av_m \times \frac{b}{(460 + t)},$$

where $b$ is the barometric pressure in inches of mercury and $t$ is the temperature of the air in the pipe in degrees Fahrenheit.

It should be noted that in this equation $b$ is the pressure of the air (in inches of mercury) in the pipe, so that the remarks made in Chapter V. regarding the allowance that must be made for the static pressure in the pipe if this exceeds 8 inches of water above or below atmosphere, apply also to cases in which the vane anemometer is used. It is, however, very unlikely that such static pressures will be experienced in any practical case in which the vane anemometer can conveniently be employed.

## The Theory of the Vane Anemometer.*

**Aerodynamic Characteristics of Flat Plates.**—The vane anemometer consists of a number of small flat plates inclined at an angle to the direction of the wind; let us therefore consider the system of forces that acts on a flat plate when exposed to an air current. In Fig. 35 AB represents an end view of the plate inclined at an angle $\alpha$ to the wind direction, this angle being usually termed the angle of incidence of the plate.

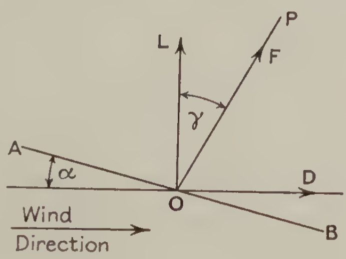



Fig. 35.

Let V be the wind speed, $\rho$ the density of the air, and S the area of the plate. Experiment shows that the aerodynamic

* Most of the material in this section has been taken from the following papers written by the author: "A Low-Speed Vane Anemometer," *Journal Scientific Instruments*, January, 1926; "The Theory of the Vane Anemometer," *Phil. Mag.*, 1926, ii., p. 881.

reactions in the plate may be reduced to a single force F acting along OP, the point O, whose position along AB depends upon the incidence $\alpha$, being known as the centre of pressure. It is found also that OP usually makes a small angle $\gamma$, as shown,

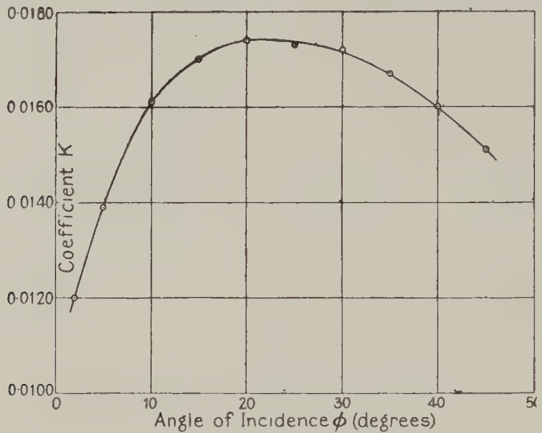



FIG. 36.

with the normal to the wind direction through O, and that the magnitude of the force F is given by the equation,

$$F = K\rho V^2 S\alpha, \quad . \quad . \quad . \quad . \quad (1)$$

where K is a numerical coefficient whose value depends upon $\alpha$. Usually it will be found more convenient to resolve the

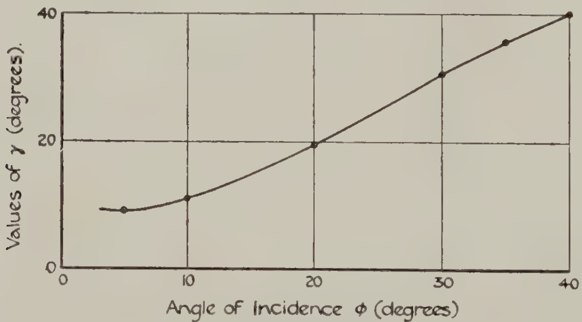



FIG. 37.

force F into two components L and D respectively perpendicular to and along the wind direction. Thus

$$\left.\begin{aligned} L &= K\rho V^2 S\alpha \cos\gamma \\ D &= K\rho V^2 S\alpha \sin\gamma \end{aligned}\right\} \quad . \quad . \quad . \quad (2)$$

Values of K at different angles of incidence have been obtained experimentally by the author for typical anemometer blades, and are given in the following table and in Fig. 36. Values of $\gamma$ deduced from G. Eiffel's experimental results * for small square flat plates are shown in the same table and are plotted in Fig. 37.

TABLE VII.

| Angle of Incidence Degrees. | K. | $\gamma$ Degrees. |
|---|---|---|
| 5 | 0·0123 | 9·0 |
| 10 | 0·0139 | 11·0 |
| 20 | 0·0174 | 19·5 |
| 30 | 0·0172 | 30·7 |
| 35 | 0·0167 | 35·8 |
| 40 | 0·0160 | 40·3 |
| 45 | 0·0151 | — |

**Application of Flat Plate Characteristics to the Anemometer.**—We may now consider the aerodynamic characteristics of flat plates in relation to their application to the vane anemometer. The following notation will be used :—

F = resultant wind force on one blade,
T = frictional resisting torque,
Q = wind torque on blades,
V = wind velocity,
$v$ = linear tangential velocity of centre of pressure of blade,
$\overline{V} = \sqrt{V^2 + v^2}$ = velocity of wind relative to blade,
K = wind force coefficient (see above),
$\rho$ = air density = $\dfrac{\text{weight per unit volume}}{g}$,
$\theta$ = inclination of blade (when at rest) to wind direction,
$\phi$ = inclination of blade to relative wind = effective incidence of blade,
$\gamma$ = inclination of resultant wind force on blade to perpendicular to direction of relative wind,
$n$ = rotational speed of blades,
N = rotational speed of pointer,
C = gearing constant, such that N = C$v$,

* See *La Resistance de l'Air et l'Aviation*, p. 134.

A = area of one blade,
D = diameter over blade tips,
$m$ = number of blades,
$r$ = distance of centre of pressure of blades from axis of rotation,
I = total moment of inertia of all blades about axis of rotation,
$g$ = acceleration due to gravity.

**Case 1—Mechanical Friction Neglected.**—Let PQ (Fig. 38) represent one blade inclined at an angle $\theta$ to the wind direction RO, and let O be the centre of pressure of the blade. It should be noted that the blade is rotating in a circular path perpendicular to the plane of the paper, and that at the instant under consideration the blade is at the top of its path and moving

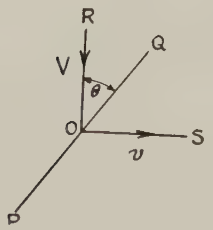



FIG. 38.

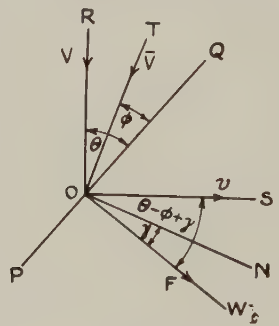



FIG. 39.

to the right with a velocity $v$ along OS, perpendicular to the wind direction. The blade speed $v$ will be such that the resultant of V and $v$ lies along QP, and there will be no resultant force perpendicular to the plane of the blade,

$$\left.\begin{array}{ll}\text{i.e.} & v = V \tan \theta \\ \text{and} & N = Cv = CV \tan \theta\end{array}\right\} \quad (3)$$

**Case 2—Mechanical Friction Included.**—The velocity diagram when mechanical friction is taken into account becomes more complicated, and it is necessary to consider the forces acting on the blade. PQ (Fig. 39) will still move along OS, perpendicular to the wind direction, but at a lower speed $v$ such that the resultant of V and $v$, i.e. the wind speed $\overline{V}$ relative to the blade, will now, when the motion has become steady, be inclined at an angle $\phi$ to the plane of the blade. The blade

thus becomes a flat plate inclined to the wind at an effective incidence $\phi$, and there will be a resultant force F on the blade acting along OW, inclined at an angle $\gamma$ to ON, the normal to the relative wind direction. The component of F along OS, multiplied by the distance of O from the axis of rotation and by the number of blades, provides the torque which overcomes the resisting torque due to mechanical friction.

If, therefore, we neglect any change in velocity of the wind during its passage through the vane circle, and any interference effect which one blade may exercise on its neighbours, we may use equation (1) to give us, for the blade under consideration, the following equation for F :—

$$F = K\rho\bar{V}^2A\phi.$$

Resolving along OS, putting $\bar{V}^2 = V^2 + v^2$, and equating the wind torque to the frictional resisting torque, we therefore have for steady motion,

$$T = K\rho\phi A(V^2 + v^2)mr\cos(\theta - \phi + \gamma). \quad . \quad . \quad (4)$$

Also, from the velocity diagram,

$$v = V\tan(\theta - \phi). \quad . \quad . \quad . \quad (5)$$

We shall find it convenient at this stage to consider the practical aspects of the equations so far derived. It will be apparent that the speed of rotation of a given set of vanes will be determined not only by the wind speed, but also by the mechanical friction introduced by the spindle bearings and also by the gearing. In the ideal case where friction is entirely absent the calibration curve of the instrument, i.e. the curve of indicated wind speed against true wind speed, would be a straight line passing through the origin—see equations (3)—whose slope would be determined (once the gearing ratio is fixed) by the inclination of the vanes to the wind direction. In practice, however, friction exercises an effect which is felt mainly at low speeds when the aerodynamic forces on the vanes are small. At high speeds, when the relative effect of friction is small, $\phi$ becomes small compared with $\theta$, and we see from equation (5) that the calibration curve of the instrument will be practically linear. At low speeds $\phi$ is comparable in magnitude with $\theta$ and $v$ will be zero at a certain low value of V when $\phi$ becomes equal to $\theta$.

Actually the behaviour of all well-made anemometers fully

substantiates the above reasoning. The usual type of calibration curve for such instruments is illustrated in Fig. 40. At high speeds the curve exhibits no sensible departure from linearity, but at low speeds it bends over somewhat in the manner shown, the abscissa OA representing the true wind speed at which the vanes just begin to turn, that is the limiting speed at which $\phi$ becomes equal to $\theta$. The magnitudes of the wind speeds corresponding to OA and to OB (above which the curve is linear) vary, as is naturally to be expected, with different instruments; in the author's sensitive low-speed anemometer, to which reference has been made above, OA was found to be about 0·6 and OB about 1·0 feet per second.

The friction torque T in equation (4) cannot be computed

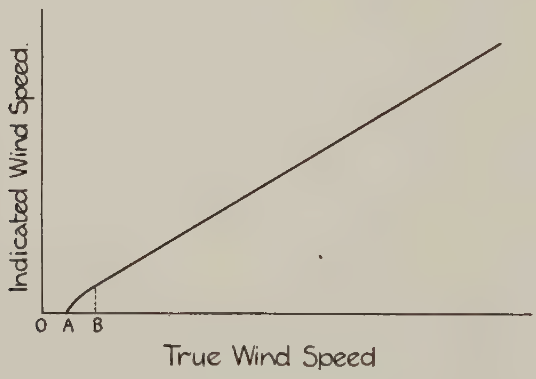



FIG. 40.—Typical calibration curve for vane anemometer.

from first principles; it will obviously depend upon the construction of the instrument, and so will have to be determined separately for each anemometer by experiment, if a knowledge of its magnitude is required. The necessary tests, however, are simple, consisting, as they do, only of a calibration to establish the relation between $v$ and V for a number of values of the wind speed V. The torque T for a given speed V can then be calculated by the use of equations (4) and (5), $v$ being taken from the calibration curve, and the values of K and $\gamma$ being obtained from Figs. 36 and 37 at the values of $\phi$ calculated from equation (5).

Now the torque T is made up of two components, one due to the weight W of the moving parts, and one due to the end thrust of the spindle on the rear bearing, arising from the

component P of the resultant wind force acting along the wind direction. Hence we may write

$$T = \mu W r_1 + \mu P r_2, \quad . \quad . \quad . \quad (6)$$

where $\mu$ is the coefficient of friction, and $r_1$ and $r_2$ are the effective radii, whose actual values do not concern us, at which the forces $\mu W$ and $\mu P$ act.

If, as is generally the case, the bearings are cylindrical, it is justifiable to regard $r_1$ and $r_2$ as constant. If, further, we assume that $\mu$ is constant, we may, since W is constant, write (6) in the form

$$T = g + hP, \quad . \quad . \quad . \quad . \quad (7)$$

where $g$ and $h$ are constants.

T is given by equation (4) and can be calculated from the calibration curve, as already shown. Similarly, P will be given by

$$P = K\phi\rho A(V^2 + v^2)\, m \sin(\theta - \phi + \gamma), \quad . \quad (8)$$

and is calculable in an analogous manner. From (7) we may expect a curve of T against P to be a straight line, from which the values of $g$ and $h$ can be determined. Values of T and P for the low-speed anemometer previously mentioned have been calculated and are plotted in Fig. 41. It will be seen that, except for the small values near the origin—corresponding to values of V below 1 foot per second—the points lie well on a straight line, which gives values of $g$ and $h$ of $1{\cdot}6 \times 10^{-6}$ and $8{\cdot}0 \times 10^{-2}$ respectively. The departure from linearity at low speeds may be due to a variety of causes which are discussed more fully below.

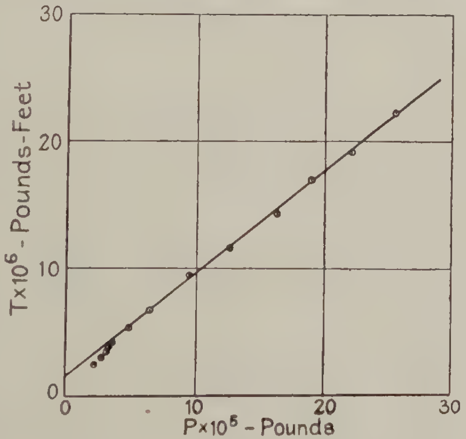



FIG. 41.

The values of T and P plotted in Fig. 41 are given in the

following table. The mechanical constants of the instrument to which they relate are as follows :—

$A = 0{\cdot}0068$ sq. ft. ; $r = 0{\cdot}115$ ft. ; $m = 8$ ; $D = 0{\cdot}312$ ft.
$\theta = 45°$ ; gear reduction (vanes to pointer) $= \frac{1}{50}$.
$I = 192 \times 10^{-8}$ slugs-ft. units.

TABLE VIII.

| Wind Speed V, Feet per Second. | $T \times 10^6$ Pounds-Feet. | $P \times 10^5$ Pounds. |
|---|---|---|
| 0·59 | 2·46 | 2·17 |
| 0·7 | 2·98 | 2·67 |
| 0·8 | 3·42 | 3·02 |
| 0·9 | 3·85 | 3·34 |
| 1·0 | 4·18 | 3·57 |
| 1·5 | 5·39 | 4·86 |
| 2·0 | 6·69 | 6·40 |
| 4·0 | 11·82 | 12·60 |
| 6·0 | 16·95 | 18·98 |
| 8·0 | 22·20 | 25·60 |

**Effect of Variations of Air Density.**—We may turn now to a consideration of the effects of variations of atmospheric density on the readings of a vane anemometer. We have already seen that these effects are likely to be mainly felt at low wind speeds, so that if an anemometer is employed for the measurement of such speeds, for which purpose it has an eminent superiority over other types of air-speed measuring instruments, errors may arise on account of the fact that when measurements are being taken the air density may differ from that existing at the time when the instrument was calibrated, for which conditions only the calibration curve is strictly valid.

The calibration curve or certificate accompanying the anemometer should, in the case of an instrument intended for low-speed work, bear a note recording the atmospheric density at the time of calibration. We shall assume also that the linear speed of the vanes $v$ is known for each value of the wind speed V, since, although the calibration will be given in terms of revolutions per second for a series of values of V, it is a simple matter to deduce $v$ from these figures and the gearing ratio and the dimensions of the instrument. Thus, if N is the rotational speed of the pointer at a wind speed V,

$$v = \frac{N}{C} = 2\pi r R n,$$

where R is the gear ratio of reduction between the vanes and the pointer.

The radius $r$ (see list of symbols, p. 111) may be taken with sufficient accuracy as the radius of the centre of area of a blade from the axis of rotation.

The problem we are now considering is the determination of the effect of a variation in $\rho$ on the value of the blade speed $v$. Let suffixes 0 refer to the conditions under which the instrument was calibrated, and suffixes 1 to the conditions under which measurements are being made, i.e. when the air density has changed from $\rho_0$ to $\rho_1$. For a given wind speed $V_0$, the linear blade speed $v_0$ can be obtained from the calibration curve when the density is $\rho_0$, and the resisting torque is given by

$$T_0 = K_0\phi_0\rho_0 A(V_0^2 + v_0^2)mr \cos(\theta - \phi_0 + \gamma_0),$$

and the end thrust by

$$P_0 = K_0\phi_0\rho_0 A(V_0^2 + v_0^2)m \sin(\theta - \phi_0 + \gamma_0).$$

Also, from (7),

$$T_0 = g + hP_0.$$

Now suppose the air density alters to $\rho_1$. At a certain value $V_1$ of the wind speed and a corresponding value $v_1$ of the blade speed, the torque $T_1$ will be the same as before.
That is,

$$T_0 = g + hP_0 = T_1 = g + hP_1,$$

from which it follows that

$$P_0 = P_1.$$

Hence

$$K_0\phi_0\rho_0 A(V_0^2 + v_0^2)mr \cos(\theta - \phi_0 + \gamma_0) = K_1\phi_1\rho_1 A(V_1^2 + v_1^2)mr \cos(\theta - \phi_1 + \gamma_1) \quad . \quad . \quad (9)$$

and

$$K_0\phi_0\rho_0 A(V_0^2 + v_0^2)m \sin(\theta - \phi_0 + \gamma_0) = K_1\phi_1\rho_1 A(V_1^2 + v_1^2)m \sin(\theta - \phi_1 + \gamma_1). \quad . \quad . \quad (10)$$

Dividing (10) by (9), we obtain

$$\tan(\theta - \phi_0 + \gamma_0) = \tan(\theta - \phi_1 + \gamma_1),$$

so that

$$\phi_0 - \gamma_0 = \phi_1 - \gamma_1. \quad . \quad . \quad . \quad (11)$$

Now $\gamma$ is a definite function of $\phi$, so that (11) can only be true if $\phi_0 = \phi_1$, and since

$$\frac{v_0}{V_0} = \tan(\theta - \phi_0),$$

and

$$\frac{v_1}{V_1} = \tan(\theta - \phi_1),$$

the equality of $\phi_0$ and $\phi_1$ leads to the relation

$$\frac{v_0}{V_0} = \frac{v_1}{V_1}. \qquad \qquad (12)$$

Also, since K is a function of $\phi$, $K_0 = K_1$. Hence equation (9) reduces to

$$\rho_0(V_0^2 + v_0^2) = \rho_1(V_1^2 + v_1^2),$$

and, putting in the condition represented by (12), it is easy to see that

$$\frac{v_1}{v_0} = \frac{V_1}{V_0} = \sqrt{\frac{\rho_0}{\rho_1}}. \qquad \qquad (13)$$

This equation indicates a ready means of deriving the calibration curve of the anemometer at density $\rho_1$ from that given at density $\rho_0$, since the interpretation of (14) is that a point $(v_0, V_0)$ on the $\rho_0$ curve becomes the point $\left(v_0\sqrt{\frac{\rho_0}{\rho_1}}, V_0\sqrt{\frac{\rho_0}{\rho_1}}\right)$ on the $\rho_1$ curve. Hence a point P′ on the new curve can be obtained from any point P on the original curve, by joining P to the origin O and producing the line OP to P′, so that $\frac{OP'}{OP} = \sqrt{\frac{\rho_0}{\rho_1}}$ (see Fig. 42).

Except at the lowest speeds, the original calibration curve will generally be a straight line whose equation may be expressed in the form

$$v = pV + q, \qquad \qquad (15)$$

where $p$ and $q$ are constants.

It is easy then to show that the corresponding portion of the calibration curve for density $\rho_1$ will be a parallel straight line whose equation will be

$$v = pV + q\sqrt{\frac{\rho_0}{\rho_1}}. \qquad \qquad (16)$$

Hence the linear portion of the new curve can be derived from that of the original curve by obtaining one point in the manner indicated above and drawing a parallel line through this point.

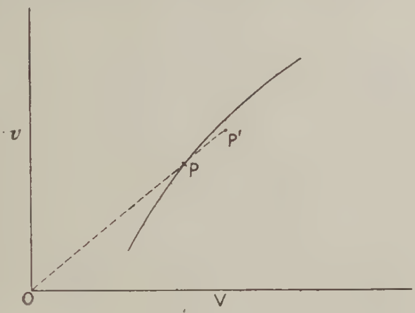


FIG. 42.

Generally, the calibration curve will not be expressed in terms of $v$, the linear blade speed, but of N, the rotational speed of the pointer. This will not, however, invalidate the above argument; its only effect will be to alter the values of the constants $p$ and $q$ in (15) and (16).

In practice it is not necessary always to draw a new calibration curve each time the value of the density changes. The air speed corresponding to an observed rotational speed can be obtained by calculation as follows. The calibration curve, obtained at density $\rho_0$, will be given in terms of the true wind speed V and an observed quantity, which we may denote by X, which will be the rotational speed of the pointer, or the "indicated air speed" if the dial is graduated in feet. In the latter case, X will simply bear a constant ratio to the rotational speed, so that the two cases are in effect identical. Suppose now that the quantity X is observed when the density is $\rho_1$. The corresponding value of X when the density is $\rho_0$ will be $X\sqrt{\frac{\rho_1}{\rho_0}}$, and this will correspond to a value V of the wind speed, as determined from the calibration curve. The value of the wind speed required can then be shown to be equal to $V\sqrt{\frac{\rho_0}{\rho_1}}$. To summarise, therefore, the procedure is to multiply the observed value of X by $\sqrt{\frac{\rho_1}{\rho_0}}$; to read off the value of V corresponding to this from the calibration curve; and finally to multiply this value of V by $\sqrt{\frac{\rho_0}{\rho_1}}$.

It will generally be found that density corrections may be neglected unless the density changes are of the order of at

least 5 or 6 per cent., except possibly for accurate work at very low speeds. Density changes of this magnitude rarely occur under ordinary conditions, but may easily be experienced if the anemometer is used under conditions of climate and altitude differing appreciably from those under which the calibration was made, or if the instrument is used to measure the flow of hot air.

The above methods of correction for density changes have been derived on the assumption that the friction torque can be expressed in the linear form of equation (7). This is probably not quite true at the lowest speeds, but the method of correction will be found even in this region to give sufficiently close approximations to actual conditions.

**Optimum Blade Angle.**—We may now consider another aspect of the relation between the frictional and aerodynamic forces acting on the vanes of an anemometer. When the wind speed is such that the vanes are on the point of commencing to rotate, the angle $\phi$ will become equal to the blade angle $\theta$, and $v$ will be zero. Hence equations (4) and (8) become

$$\mathrm{T} = \mathrm{K}\theta\rho\mathrm{AV}^2 mr \cos\gamma,$$

and

$$\mathrm{P} = \mathrm{K}\theta\rho\mathrm{AV}^2 m \sin\gamma,$$

so that, writing T in the form of equation (7),

$$\mathrm{K}\theta\rho\mathrm{AV}^2 mr \cos\gamma = g + h\mathrm{K}\theta\rho\mathrm{AV}^2 m \sin\gamma,$$

or

$$\rho\mathrm{A}m\mathrm{V}^2(\mathrm{K}\theta r \cos\gamma - h\mathrm{K}\theta \sin\gamma) = g. \qquad (17)$$

We may use this equation to determine theoretically the angle at which the vanes of an anemometer must be set in order that they may commence to rotate at as low a speed as possible. For, since $g$ is constant, the left-hand side of (17) is constant, and if V is to be a minimum it follows that $\mathrm{K}\theta(r \cos\gamma - h \sin\gamma)$ must be a maximum.

A complication is introduced by the fact that the departure from linearity of the curve of Fig. 41 at low speeds shows that $h$ cannot be taken as the slope of the linear portion when we are considering, as is the case at present, conditions in the neighbourhood of zero rotational speed. The variation from the straight line law of equation (7) between friction torque and end thrust at very low rates of rotation may be due to a change of the coefficient of friction $\mu$ in this region. Little is known about the coefficients of friction between solid surfaces

moving at very low relative speeds, but there seems to be some evidence, slight though it is, that the value of $\mu$ between surfaces at rest may be different from that when the surfaces are moving relatively to one another. Further, for all except very slow relative motions $\mu$ is found to be approximately constant, and independent of the speed. It is, then, possible that in the transition stage from rest to motion $\mu$ changes rapidly but continuously from its constant static value to its constant dynamic value. If this is so, when the vanes are just commencing to rotate, $\mu$ is probably not the same as it is when they are in steady motion, and this will furnish one possible cause for the deviation of the low-speed points in Fig. 41 from the line through the other points. Other causes that may conceivably contribute to this departure are variations in the type of flow of the air past the blades that may occur at very low wind speeds, and the interference between the blades that may tend to become more pronounced as the rotational speed becomes smaller in relation to the wind speed, that is, as the effective incidence $\phi$ of the blades becomes larger.

Whilst, therefore, any application of equation (17) to determine the optimum blade angle cannot, in the present state of knowledge, be regarded as rigorous, some useful indications are nevertheless supplied. If we attribute the variation in the straight line law between T and P at low wind speeds entirely to changes in $\mu$, we may proceed as follows. Let us write $\mu_{\max}$ for the constant value of $\mu$ at the higher speeds, and $\mu_{\min}$ for its value when the vanes are just commencing to rotate. Since $\mu$ is no longer to be taken as constant, equation (6) must now be written in the form,

$$\mathrm{T} = \mu(g' + h'\mathrm{P}), \qquad (18)$$

where $g'$ and $h'$ are constants.

If $\mathrm{T}_{\min}$ and $\mathrm{P}_{\min}$ are the values of T and P when the vanes are just on the point of commencing to rotate, equation (18) becomes

$$\mathrm{T}_{\min} = \mu_{\min}(g' + h'\mathrm{P}_{\min}), \qquad (19)$$

and, at the higher speeds, we have

$$\mathrm{T} = \mu_{\max}(g' + h'\mathrm{P}),$$

where $\quad g'\mu_{\max} = g$ in equation (7),

and $\quad h'\mu_{\max} = h$ in equation (7).

From (19) $$\mu_{min} = \frac{T_{min}}{g' + h'P_{min}},$$

so that $$h'\mu_{min} = \frac{h'T_{min}}{g' + h'P_{min}}.$$

Multiplying and dividing by $\mu_{max}$, we have, therefore,

$$h'\mu_{min} = \frac{h'\mu_{max}T_{min}}{g'\mu_{max} + h'\mu_{max}P_{min}}. \quad . \quad . \quad (20)$$

Now $h'\mu_{max}$ and $g'\mu_{max}$ are, as we have seen, equal to $h$ and $g$ respectively in equation (7), that is, they can be obtained by plotting T against P as shown in Fig. 41. Also $T_{min}$ and $P_{min}$ are the values of the torque and end thrust when the vanes are just commencing to rotate, and can be calculated by putting, in equations (4) and (8), $\phi = 0$ and $v = 0$, and reading off the value of V at which the calibration curve between $v$ and V cuts the V-axis. Hence $h'\mu_{min}$ can be calculated by the use of (20)

Returning now to equation (17), we see that for the best blade angle $K\theta(r \cos \gamma - h'\mu_{min} \sin \gamma)$ must be a maximum, the quantity $h'\mu_{min}$ being equal to the value of $h$ when the vanes are on the point of rotating. At this value of the rotational speed, namely, zero, the angle of incidence of the blades to the wind is given by $\theta$, their geometrical incidence, so that K and $\gamma$ can be obtained directly for a given value of $\theta$ from the data of Figs. 36 and 37. The quantity

$$K\theta(r \cos \gamma - h'\mu_{min} \sin \gamma)$$

can thus be evaluated for different assumed values of $\theta$, and will be found to exhibit a maximum value for a definite value of $\theta$. This angle will therefore be the angle at which the vanes should be set in order that they may rotate at the lowest possible speed.

For the author's low-speed anemometer we have already seen that $h'\mu_{max} = h = 8\cdot0 \times 10^{-2}$ and $g'\mu_{max} = g = 1\cdot6 \times 10^{-6}$. Also, from Table VIII., $T_{min} = 2\cdot46 \times 10^{-6}$ and $P_{min} = 2\cdot17 \times 10^{-5}$. Hence, from (20), $h'\mu_{min}$ will be found to be equal to 0·059. For this instrument, also, $r = 0\cdot115$ feet, so that the best blade angle for this type of anemometer is that at which

$$K\theta(0\cdot115 \cos \gamma - 0\cdot059 \sin \gamma)$$

is a maximum. In Fig. 43 this quantity is plotted against $\theta$, and it will be seen that the maximum value occurs at $\theta = 31°$,

which is therefore the best blade angle as indicated by the theory.

Actually, from experiments made by the author, the best blade angle for an anemometer appears to be about 40°. In these experiments, a series of tests of the minimum wind speed at which the vanes would rotate was made with an instrument whose vanes were successively adjusted to each of a number of angles of incidence. In view of the remarks made above on the uncertainty regarding the variation of frictional and interference effects that exists at these very low speeds, the discrepancy of about 9° between the theoretical and experimental optimum values of $\theta$ is not regarded as excessive.

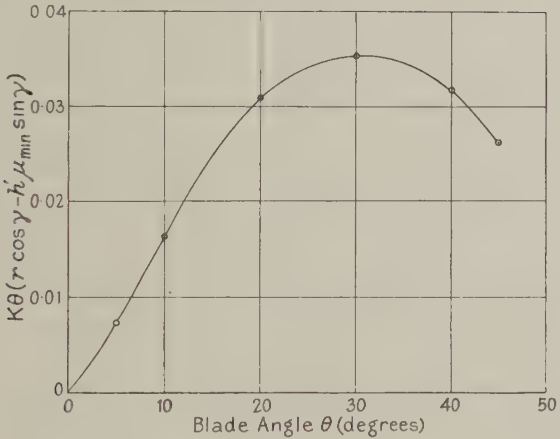



FIG. 43.

The instrument on which the tests were made was fairly représentative of vane anemometers in general, and it may be taken that in all such instruments the blades should be set at 40°. The author's low-speed anemometer was designed before the theory was developed, and its blade angle of 45° was fixed arbitrarily. A similar instrument subsequently constructed with a 40° vane angle was found to commence rotation at a wind speed definitely lower than that at which motion of the original anemometer commenced.

**Effect of a Fluctuating Wind.** — We have tacitly assumed in the preceding analysis that the anemometer is placed in a steady wind whose speed is constant and uniform across the whole vane circle. In practice, however, the wind

speed is usually both more or less unsteady in magnitude and variable from point to point across the vanes, and it is necessary to consider what errors may be incurred from each of these causes. We will deal firstly with the effect of a fluctuating wind speed, which we will assume at any instant to be the same at all points over the vane circle, but to vary with time over the whole circle. We must exclude from consideration natural winds, and confine our attention to those cases in which an anemometer is used to measure the speed of an artificially produced air current. Such a current will usually be obtained by means of a fan running at a speed which is intended to be constant, but which will probably fluctuate about a mean value. As a consequence the wind speed will also fluctuate in a more or less periodic manner about a mean value, and for the purpose of analysis we will assume that the wind speed V at any time $t$ is given by the simple harmonic relation,

$$V = V_0(1 + \lambda \sin pt), \quad . \quad . \quad . \quad (21)$$

where $V_0$ is the value of V at any time $t = 0$.

It should be noted that in this equation $\lambda$ is the amplitude of the fluctuation, $V_0$ is the mean speed about which fluctuations occur, and that $\frac{2\pi}{p}$ is the time for one complete cycle.

Now the aerodynamic theory of airscrews has established the fact that both for an airscrew producing a wind, and for a windmill driven by the wind, the wind torque Q is given by an equation of the form

$$\frac{Q}{\rho V^2 D^3} = f\left(\frac{nD}{V}\right), \quad . \quad . \quad . \quad (22)$$

where $f\left(\frac{nD}{V}\right)$ represents some function of $\left(\frac{nD}{V}\right)$. We may therefore assume that an equation of this type holds for a vane anemometer. When the anemometer is in steady motion $Q = -T$, and therefore by plotting $-\frac{T}{\rho V^2 D^3}$ against $\frac{nD}{V}$ the form of the function $f\left(\frac{nD}{V}\right)$ can be determined. To take a concrete case, we may consider once more the low-speed anemometer,*

* Continual reference to this anemometer is necessary, since it is the only instrument whose complete characteristics are available to the author.

for which the values of T have been calculated (see Table VIII.). Values of Q (= — T) are plotted against $\frac{nD}{V}$ in Fig. 44, and it will be seen that all the points for values of V = 1·0 and upwards lie on a straight line. It should be noted that this range of V corresponds with the linear portion of the calibration curve (See Errata slip)

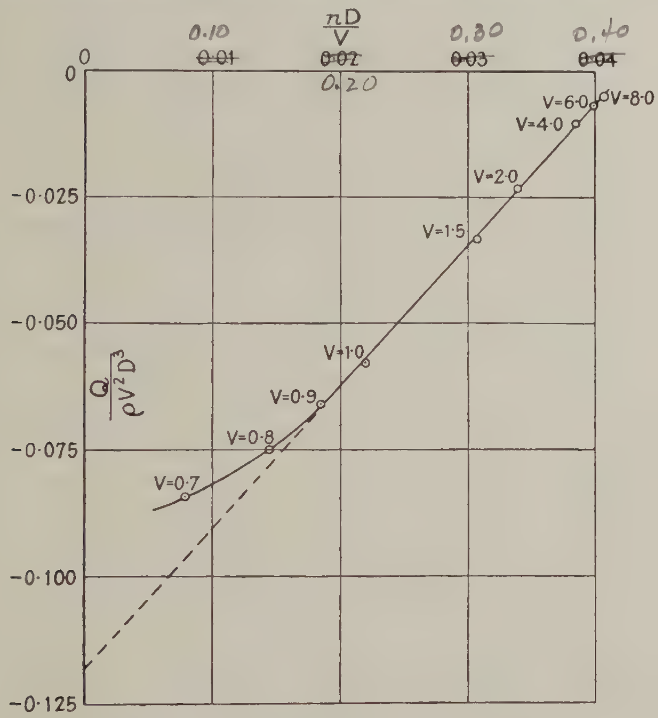



FIG. 44.

of the instrument, so that it appears that over this region we may write

$$\frac{Q}{\rho V^2 D^3} = a + b\frac{nD}{V}, \quad . \quad . \quad . \quad (23)$$

where $a$ and $b$ are constants, and this result may be expected to apply, over the linear range, to all vane anemometers of normal design, the values of $a$ and $b$ varying from one to another. For the particular instrument under consideration the values of $a$ and $b$ taken from Fig. 44 are — 0·118 and + 0·383 respectively.

Let us now imagine the fluctuating wind speed given by equation (21) suddenly impressed upon an anemometer which was previously rotating at a steady rate $n_0$ in a steady wind of speed $V_0$.

From equation (13) we have, at time $t = 0$,

$$Q_0 = \rho V_0{}^2 D^3\left(a + b\frac{n_0 D}{V}\right),$$

and at a time $t$ shortly after

$$Q_1 = Q_0 + q = \rho(V_0 + \bar{v})^2 D^3\left\{a + b\frac{n_0 + n}{V_0 + \bar{v}}D\right\},$$

where $q$, $\bar{v}$, and $n$ are small changes in $Q_0$, $V_0$ and $n_0$ respectively. Hence, by subtraction,

$$\frac{q}{\rho D^3} = 2aV_0\bar{v} + a\bar{v}^2 + bD(V_0 + \bar{v})(n_0 + n) - bDn_0V_0$$
$$= bD(V_0 + \bar{v})n + \bar{v}(2aV_0 + bDn_0) + a\bar{v}^2.$$

Now $\bar{v}$ is the increase in wind speed, and is given by $\lambda V_0 \sin pt$, so that, substituting this value of $\bar{v}$ in the above equation, we have

$$\frac{q}{\rho D^3} = bDV_0(1 + \lambda \sin pt)n + \lambda V_0(2aV_0 + bDn_0)\sin pt$$
$$+ a\lambda^2 V_0{}^2 \sin^2 pt.$$

Since $q$ is the excess wind torque acting on the instrument, which produces an increase $n$ in the rotational speed, the equation of motion, which is given by

$$I\frac{d}{dt}(\text{angular velocity}) + q = 0,$$

becomes, since the angular velocity produced by $q$ is equal to $2\pi n$,

$$\frac{2\pi I}{\rho D^3}\frac{dn}{dt} + bDV_0(1 + \lambda \sin pt)n = -(2aV_0 + bDn_0)\lambda V_0 \sin pt$$
$$- a\lambda^2 V_0{}^2 \sin^2 pt,$$

or $$\frac{dn}{dt} + \alpha(1 + \lambda \sin pt)n = \beta \sin pt + \delta \sin^2 pt, \quad . \quad (24)$$

$$\left.\begin{aligned} \text{where} \qquad \alpha &= \frac{b\mathrm{DV}_0\rho\mathrm{D}^3}{2\pi\mathrm{I}}, \\ \beta &= -\frac{\lambda\mathrm{V}_0(2a\mathrm{V}_0 + b\mathrm{D}n_0)\rho\mathrm{D}^3}{2\pi\mathrm{I}}, \\ \text{and} \qquad \delta &= -\frac{a\lambda^2\mathrm{V}_0{}^2\rho\mathrm{D}^3}{2\pi\mathrm{I}}. \end{aligned}\right\} \quad . \quad . \quad (24a)$$

Equation (24) is of the standard form $\frac{dy}{dx} + \mathrm{P}y = \mathrm{Q}$, where P and Q are functions of $x$, and the solution is therefore

$$ne^{\int\alpha(1+\lambda\sin pt)dt} = \int(\beta\sin pt + \delta\sin^2 pt)e^{\int\alpha(1+\lambda\sin pt)dt}\,dt + \mathrm{C},$$

$$\text{or } ne^{\alpha(t-\frac{\lambda}{p}\cos pt)} = \int(\beta\sin pt + \delta\sin^2 pt)e^{\alpha(t-\frac{\lambda}{p}\cos pt)}dt + \mathrm{C}, \qquad (25)$$

where C is a constant.

This solution cannot be reduced further without making certain assumptions based on practical considerations. We have assumed that the wind speed varies according to a sine law about a mean value V, but we have not yet assigned magnitudes to the amplitude or period of the variation.

For the purpose of illustration we will assume that the frequency is 2 per second (i.e. the periodic time is $\frac{1}{2}$ second) and the amplitude $\pm$ 10 per cent. of the mean speed. These assumptions are more or less representative of many working conditions. Hence in equation (21) we have

$$p = \frac{2\pi}{\frac{1}{2}} \text{ and } \lambda = 0{\cdot}1,$$

$$\text{i.e.} \qquad \frac{\lambda}{p} = 0{\cdot}008.$$

Equation (25) then becomes

$$ne^{\alpha(t-0\cdot008\cos pt)} = \int(b\sin pt + \delta\sin^2 pt)e^{\alpha(t-0\cdot008\cos pt)}dt + \mathrm{C}.$$

Now the maximum value of 0·008 cos $pt$ is 0·008, so that when $t = 0{\cdot}8$, i.e. before the motion has proceeded for one second, this term becomes less than 1 per cent. of $t$, and afterwards becomes relatively smaller. It is therefore legitimate to

neglect 0·008 cos $pt$ in comparison with $t$ and we may write, with sufficient accuracy,

$$ne^{at} = \int(\beta \sin pt + \delta \sin^2 pt)e^{at}dt + C,$$

the right-hand side of which can now be integrated, giving, after reduction,

$$n = \frac{\beta(a \sin pt - p \cos pt)}{a^2 + p^2} + \frac{\delta(a^2 + 4p^2 - a^2 \cos 2pt - 2ap \sin 2pt)}{2a(a^2 + 4p^2)} + Ce^{-at} . \quad . \quad (26)$$

The value of C can be obtained by putting $n = 0$ when $t = 0$, since $n$ is the change in the rotational speed of the vanes from its value at the commencement of the fluctuating motion, and in this way we arrive at the complete solution,

$$n = \frac{\beta(a \sin pt - p \cos pt)}{a^2 + p^2} + \frac{\delta(a^2 + 4p^2 - a^2 \cos 2pt - 2ap \sin 2pt)}{2a(a^2 + 4p^2)} + pe^{-at}\left[\frac{\beta}{a^2 + p^2} - \frac{2\delta p}{a(a^2 + 4p^2)}\right]. \quad . \quad (27)$$

If we examine this solution we find that the motion may be regarded as composed of two parts, one a steady periodic motion represented by the sum of the first two terms, and the other, represented by the term $Ce^{-at}$, a damped motion which has its maximum value when $t = 0$ and decays rapidly as the time proceeds. In the case of the low-speed anemometer we find from (24$a$), by inserting the values of the mechanical constants already given, that $a = 0{\cdot}718V_0$, so that if we take the case when $V_0 = 1$ foot per second, the damped part of the motion is equal to $Ce^{-0{\cdot}718t}$. Now $e^{-0{\cdot}718t}$ becomes equal to 0·1 when $t = 3{\cdot}2$ approximately, so that the damped motion is reduced to one-tenth of its initial amplitude in about $3\frac{1}{5}$ seconds, and it will be seen that this time will decrease as V increases. It appears therefore that the motion represented by (27) rapidly assumes a steady periodic character given by the sum of the first two terms, and it can easily be shown that the average

value of $n$ is $\frac{\delta}{2\alpha}$, or, when the values of $\delta$ and $\alpha$ from (24*a*) are inserted, $-\frac{a\lambda^2}{2b\mathrm{D}}\mathrm{V}_0$.

In other words, this means that the anemometer, when placed in a wind fluctuating in the assumed manner about a mean value $\mathrm{V}_0$, will rotate at an average rate higher by an amount $-\frac{a\lambda^2}{2b\mathrm{D}}\mathrm{V}_0$ (higher, since $a$ is negative) than the rate at which it would rotate in a steady wind of speed $\mathrm{V}_0$; i.e. an anemometer in a fluctuating wind gives readings in excess of the mean speed.

It will be seen that the quantity $-\frac{a\lambda^2}{2b\mathrm{D}_0}$ is independent of the period of the fluctuations in the wind speed and of the moment of inertia of the vanes. It appears that these quantities will merely determine the lag of the periodic motion of the vanes behind that of the wind, whilst the mean amplitude of the motion of any given set of vanes is affected only by the amplitude of the variations of the wind speed. An inspection of (27), however, shows that the *actual* value of $n$ at any time is influenced by $p$ and I; it is only the *average* value of $n$ that is independent of these quantities.

To obtain some idea of the magnitude of the excess reading obtained in a fluctuating wind, we may apply this result to the low-speed anemometer. The values of $a$ and $b$ have already been given as $-0{\cdot}118$ and $0{\cdot}383$ respectively, and the value of D will be found in the list of constants for this instrument. The value of $-\frac{a\lambda^2}{2b\mathrm{D}}\mathrm{V}_0$ will be found to become $0{\cdot}00493\mathrm{V}_0$, on the assumption, as before, that $\lambda = 0{\cdot}1$. We can now draw up the following table in which the value of $0{\cdot}00493\mathrm{V}_0$ is shown for various values of $\mathrm{V}_0$; the value of $n_0$ corresponding to each value of $\mathrm{V}_0$ was obtained from the calibration curve of the instrument obtained in a steady wind, and the last column, in which $0{\cdot}00493\mathrm{V}_0$ is expressed as a percentage of $n_0$, shows the percentage increase in the readings of the instrument when placed in the fluctuating wind of mean speed $\mathrm{V}_0$.

It will be seen that for the conditions assumed the percentage error on the mean speed is less than 1 per cent. If, therefore, the maximum departure of the wind speed from its mean value does not exceed 10 per cent., and the frequency of the fluctua-

TABLE IX.

| $V_0$ Feet per Second. | $0{\cdot}00493\ V_0$ Revs. per Second. | $n_0$ Revs. per Second. | Percentage Increase in Reading of Anemometer. |
|---|---|---|---|
| 1 | 0·00493 | 0·70 | 0·70 |
| 2 | 0·00986 | 2·16 | 0·46 |
| 4 | 0·0197 | 4·92 | 0·40 |
| 6 | 0·0295 | 7·53 | 0·39 |
| 8 | 0·0394 | 10·42 | 0·38 |

tions in wind speed is not greater than 2 per second, the errors in the mean speed as indicated by an anemometer may be disregarded. It will in many cases be found that unsteadiness in the air current is not worse than that here assumed. In adverse circumstances, however, the maximum amplitude of the fluctuations may be as much as 50 per cent. ($\lambda = 0{\cdot}5$) or more in excess of the mean speed, although it is unlikely that the frequency of the fluctuations will be much higher than 2. Such extremely unsteady conditions sometimes occur in mine drifts. Since the error in the readings of the anemometer have been shown to be proportional to $\lambda^2$, they will be increased 25 times if $\lambda$ is 0·5 instead of 0·1, so that under such unfavourable conditions the readings may be 10 per cent. high, or even more.

**Effect of a Variation in Wind Speed Across the Vane Circle.**—In general the distribution of velocity across a pipe along which air is flowing is not uniform, the velocity being higher at the centre than near the walls. We have seen in Chapter V. that, in order to determine the quantity of air flowing in such a case, the velocity has to be measured at certain definite radii, which are the mean radii of a number of imaginary concentric zones of equal area into which the pipe has to be divided in making measurements. The question therefore arises whether, when measurements are made by means of an anemometer, the instrument must be placed with its axis at

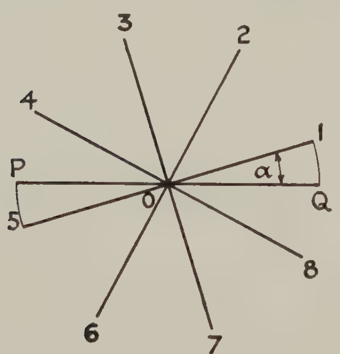



Fig. 45.

the radii in question or at other points. The answer depends on whether or not the blades will rotate at the speed appropriate to the velocity of the filament of air passing the centre of the anemometer disc, or at a rate corresponding to some other wind speed.

In Fig. 45 let O1, O2, O3, etc., represent the median lines of the vanes of an eight-bladed anemometer, and let $r$ be the radius to the centre of pressure of a blade. Let blade O1 be at an angle $\alpha$ to the reference line PQ, and let the velocity vary along PQ according to the relation

$$V = V_0(1 + Ax + Bx^2), \quad . \quad . \quad . \quad (28)$$

where V is the velocity at any point along PQ, distant $x$ from P where the velocity is $V_0$.

Also let $$PO = OQ = r.$$

Let $x_1$, $x_2$, etc., be the distances measured parallel to PQ of the centres of pressure of the blades O1, O2, etc., from O.

Then

$$x_1 = r(1 + \cos\alpha),$$

$$x_2 = r\left(1 + \cos\overline{\frac{\pi}{4} + \alpha}\right),$$

$$x_3 = r\left(1 + \cos\overline{\frac{2\pi}{4} + \alpha}\right) = r(1 - \sin\alpha),$$

and $$x_4 = r\left(1 + \cos\overline{\frac{3\pi}{4} + \alpha}\right) = r\left(1 - \sin\overline{\frac{\pi}{4} + \alpha}\right);$$

similarly $$x_5 = r(1 - \cos\alpha),$$

$$x_6 = r\left(1 - \cos\overline{\frac{\pi}{4} + \alpha}\right),$$

$$x_7 = r(1 + \sin\alpha),$$

and $$x_8 = r\left(1 + \sin\overline{\frac{\pi}{4} + \alpha}\right).$$

In order that the problem may not become unduly complicated, we shall assume that the velocity distribution perpendicular to PQ is uniform, i.e. that the velocity at all points on a line intersecting PQ and perpendicular to it is the same, and equal to the velocity at the point of intersection. In this case the velocities $V_1$, $V_2$, etc., of the air impinging on the centre

of pressure of each blade depend only upon the distances $x_1$, $x_2$, etc., and are given by the following relationships :—

$$\left.\begin{aligned}
V_1 &= V_0[1 + Ar(1 + \cos\alpha) + Br^2(1 + \cos\alpha)^2],\\
V_2 &= V_0\left[1 + Ar\left(1 + \cos\overline{\frac{\pi}{4} + \alpha}\right) + Br^2\left(1 + \cos\overline{\frac{\pi}{4} + \alpha}\right)^2\right],\\
V_3 &= V_0[1 + Ar(1 - \sin\alpha) + Br^2(1 - \sin\alpha)^2],\\
V_4 &= V_0\left[1 + Ar\left(1 - \sin\overline{\frac{\pi}{4} + \alpha}\right) + Br^2\left(1 - \sin\overline{\frac{\pi}{4} + \alpha}\right)^2\right],\\
V_5 &= V_0[1 + Ar(1 - \cos\alpha) + Br^2(1 - \cos\alpha)^2],\\
V_6 &= V_0\left[1 + Ar\left(1 - \cos\overline{\frac{\pi}{4} + \alpha}\right) + Br^2\left(1 - \cos\overline{\frac{\pi}{4} + \alpha}\right)^2\right],\\
V_7 &= V_0[1 + Ar(1 + \sin\alpha) + Br^2(1 + \sin\alpha)^2],\\
&\text{and}\\
V_8 &= V_0\left[1 + Ar\left(1 + \sin\overline{\frac{\pi}{4} + \alpha}\right) + Br^2\left(1 + \sin\overline{\frac{\pi}{4} + \alpha}\right)^2\right].
\end{aligned}\right\} \qquad (29)$$

Let the centres of pressure of the blades move, under the effect of this variable velocity, at a tangential linear speed $\bar{v}$ which would correspond, according to the calibration curve, to a wind speed $\bar{V}$. Then $\bar{V}$ and $\bar{v}$ will be such that the torque on the whole instrument rotating at this speed will be equal to the sum of the individual torques contributed by each blade.

Now we have already shown (equation (4)) that

$$\text{Torque} = K\rho A\phi(V^2 + v^2)mr\cos(\theta - \phi + \gamma).$$

Provided that the wind speed is such that the anemometer is rotating at a fair speed, i.e. that we are working on a portion of the calibration curve some distance from B in Fig. 40, no appreciable error will be introduced by assuming that $\phi$, and hence also K and $\gamma$, are constant for all the blades, so that we may write

$$\text{Torque} = K'm(V^2 + v^2).$$

The condition that the total torque is equal to the sum of the torques of all the blades is, therefore, expressed by the equation

$$8K'(\bar{V}^2 + \bar{v}^2) = K'\{(V_1^2 + \bar{v}^2) + (V_2^2 + \bar{v}^2) + \dots + (V_8^2 + \bar{v}^2)\},$$

i.e.

$$8\bar{V}^2 = V_1^2 + V_2^2 + V_3^2 + \dots + V_8^2 = \Sigma V_1^2.$$

From equations (29) it will be found, after reduction, that

$$\Sigma V_1^2 = 8V_0^2[(1 + Ar + Br^2)^2 + \frac{r^2}{8}(4A^2 + 8B + 24ABr + 27B^2r^2)],$$

which is independent of $\alpha$ and hence constant for all positions of the blades, so that

$$\bar{V} = V_0\sqrt{(1+Ar+Br^2)^2+\frac{r^2}{8}(4A^2+8B+24ABr+27B^2r^2)}. \quad (30)$$

In this equation $\bar{V}$ is, as we have seen, the wind speed indicated by the anemometer. If the instrument indicated the speed of the wind passing through the centre of its disc, the value of $\bar{V}$ would be $V_0(1 + Ar + Br^2)$, so that it appears that the indicated wind speed is the speed of the air at some point along the radius of the anemometer.

To get some idea of the magnitude of the discrepancy, we must take numerical examples. Let us assume, in order to fix our ideas, that the wind speed given by equation (28) is $V_0$ at P($x = 0$ in Fig. 45), and rises to $2V_0$ at Q ($x = 2r$), and that at $x = r$, the centre of the disc, $V = 1{\cdot}8V_0$. We can now calculate the values of A and B, and, by inserting these in equation (20), we get

$$\bar{V} = 1{\cdot}69V_0,$$

which is 6 per cent. less than the velocity at the centre of the disc, and is, in fact, the velocity at the point $x = 0{\cdot}81r$, instead of at $x = r$.

If the diameter of the anemometer is small compared with that of the pipe, it will probably be sufficiently accurate to assume a linear variation of velocity across the disc of the form

$$V = V_0(1 + Ar).$$

In this case, similar reasoning leads to the result

$$\bar{V} = V_0\sqrt{(1 + Ar)^2 + \frac{A^2r^2}{2}},$$

and, on the assumption, as before, that the velocity at $x = 2r$ is twice the velocity at $x = 0$, we have

$$\bar{V} = 1{\cdot}54V_0 = \text{velocity at the point } x = 1{\cdot}08\,r;$$

i.e. $\bar{V}$ is $2\frac{1}{2}$ per cent. greater than the velocity at the centre of the disc.

In practice, provided that the pipe diameter is reasonably large in relation to that of the anemometer, a distribution of velocity such as that assumed in both the above cases, in which the wind speed at the centre of pressure of a blade in one of its extreme positions is double its value in the other, is probably of rare occurrence, so that in general the errors in indicated wind speed will be less than those given above. Nevertheless, it is important to observe that these errors may in many cases become appreciable, and it is for this reason that it is not advisable to use an anemometer in a pipe whose diameter is less than about 6 times that of the instrument, unless it is known, either from a preliminary exploration or otherwise, that the distribution of velocity across the section of measurement is reasonably uniform.

# CHAPTER VIII.

## MISCELLANEOUS METHODS OF FLOW MEASUREMENT DEPENDING ON PRESSURE OBSERVATIONS.

**Measurement of Flow in Small Pipes.**—The Pitot-static tube of the standard type is not suitable for traversing a pipe of less than about 6 inches diameter. It will be recalled that when this instrument is used to determine the volume, observations must be made at points whose positions are fixed in accordance with definite rules; for example, if six readings are taken along a diameter, one observation has to be made at a distance of 0·044 pipe diameters from each wall. In a 6-inch pipe this means that the axis of the Pitot tube must be placed at a distance from the wall of just over $\frac{1}{4}$ inch, a position in which the outside of the standard Pitot head will only be about $\frac{1}{10}$ inch from the pipe wall. In pipes of less than 6 inches diameter the outside of the standard Pitot tube will be correspondingly closer to the walls of the pipe, and the flow there is likely to be seriously influenced by the presence of the Pitot, so that errors, probably mainly on the static pressure reading, may be expected.

FIG. 46.—Small Pitot-static tube.

Some alternative to the standard Pitot tube is therefore required for small pipes. The obvious method is to use a Pitot tube of considerably smaller dimensions, and satisfactory results have been obtained in this manner by the use of an instrument of the form shown in Fig. 46. Since for such small tubes the concentric construc-

tion of the total head and static tubes presents some difficulty, the two tubes are here arranged side by side except for a short length in the head itself where the total head tube enters the static tube through the side, the junction being made air-tight by soldering. The mouth of the total head tube is about 0·03 inch in diameter, whilst the bore of the static tube is about 0·06 inch and the diameter of the static holes may be about 0·01 inch. An instrument of this kind must be calibrated before use, since it cannot easily be made geometrically similar to the standard Pitot-static tube.

**The Total Head Tube.**—The Pitot-static tube shown in Fig. 46 is an instrument requiring somewhat delicate workmanship in its construction. An alternative method of measuring the flow is to use a total head tube only, and to measure

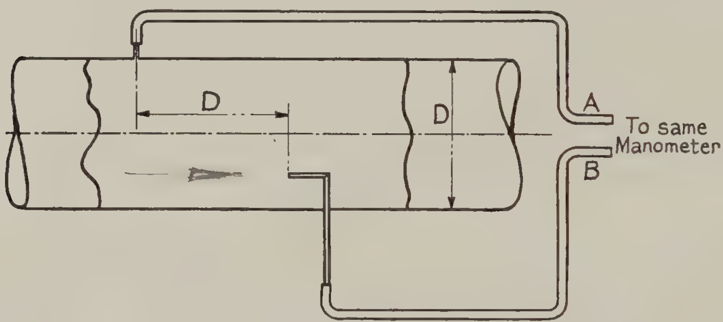



Fig. 47.—Measurement of flow with total head tube and side hole.

the static pressure separately. The advantage of this method lies in the fact that any tube with an open end facing the air current will measure total head accurately, no special shaping of the end in the stream being necessary. The outside diameter of this tube should not be greater than $\frac{1}{10}$ inch for measurements in a 6-inch diameter pipe, and should be even less for very small pipes. As regards the measurement of the static pressure, if the flow in the pipe is everywhere parallel to the walls, the static pressure is equal at all points in a section, and, further, it is found that a manometer connected to a small tube let in to the pipe so that its inner end is flush with the inside surface of the pipe, measures this pressure accurately. When, therefore, the flow is sensibly parallel, the velocity at a point in a pipe section can be determined by the arrangement shown in Fig. 47, in which the pressure leads A and B are connected to

opposite sides of the same manometer, one side of which will be under the static pressure only, whilst acting upon the other there will be the sum of the velocity head and the static pressure. The resultant reading of the manometer will therefore be the velocity head, and in order to measure the mean rate of flow the total head tube is traversed exactly as if it were a combined Pitot-static instrument. It will be observed that the total head tube in the diagram is traversed across a section situated about one pipe diameter downstream of the side tube, in order that the readings of static pressure may not be affected by the presence of the mouth of the total head tube when the observations are being taken near the pipe wall. The difference between the static pressures at these two sections one pipe diameter apart will never be sufficiently great to lead to any perceptible error in the measurement of velocity. It must be emphasised that the static pressure side connection to the

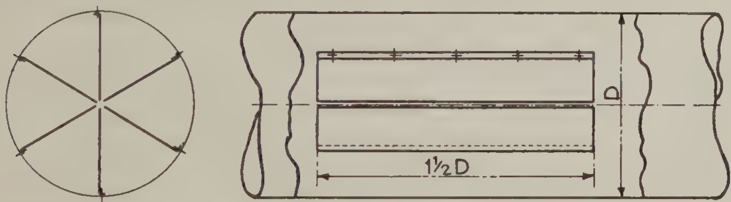



FIG. 48.—Flow straightening device.

pipe must be flush and smooth on the inside surface ; all burrs made in drilling the hole in the pipe wall to take the small side tube must be carefully removed. The bore of the side tube should not exceed about $\frac{1}{8}$ inch.

If the flow at the section is not everywhere parallel to the axis of the pipe, the static pressure at the wall as measured by the side tube will not necessarily be the same as the mean static pressure across the section. Any rotary motion of the airstream, for example, will, on account of radial accelerations, cause the static pressure at the side hole to be greater than it is at the axis. The simplest method of straightening the flow sufficiently to make the static pressure at the section sensibly uniform is to introduce, some three or four, or preferably six, diameters upstream of the section, a flow straightening device, which may consist of six thin flat radial vanes reaching from the centre of the pipe to its circumference—see Fig. 48—and about one to one and a half pipe diameters in length.

This method will be found efficacious in most cases, except possibly if a sharp bend precedes the straightener. If this is so, and it is not possible to make measurements at some other section where conditions are more favourable, or to use the small Pitot and static tube, the best that can be done is to use in place of the single side tube for measuring static pressure, four or six such tubes equally spaced round the pipe circumference, and to lead the connections from these to a common air-tight reservoir, in which the static pressure will probably be somewhere about the mean static pressure at the section. This reservoir may consist of a glass bottle with a neck sufficiently wide to take a large cork through which the necessary number of pressure connections pass; the pressure in the bottle is conducted through an additional connection to the manometer.*

It will be evident that this method does not give the true velocity at each point at which the mouth of the total head tube is placed, even if the pressure in the reservoir is actually equal to the mean of the static pressures transmitted from the side tubes in the pipe. As, in addition, there seems to be some reason to doubt whether the pressure in the reservoir is exactly equal to this mean static pressure, it will be clear that this method of measurement is not one of the highest accuracy, although for most practical purposes it will give a sufficiently close approximation to the true rate of flow. When the velocity and static pressure both vary considerably across the section, the highest precision can only be obtained by a thorough exploration by means of an instrument such as the small Pitot and static tube already described.

**The Shaped Nozzle.**—Perhaps the most convenient practical method of measuring the mean flow in a small pipe is to introduce into it a shaped nozzle of the standard German type, or any other nozzle or orifice plate for which the coefficient is accurately known, and to measure the resultant pressure drop. The quantity flowing can then be calculated from the equations already given. This method is quite suitable in cases where a sufficient length of parallel pipe exists, upstream of the section at which the nozzle or orifice is inserted, for the flow at that section to have assumed the normal distribution for pipe flow.

Very often it is not possible to realise this condition, but

* For the method of rendering this cork air-tight, see page 168.

in this case the same method may be used with a slight modification whereby advantage is taken of the uniform velocity of the jet leaving a shaped nozzle—a feature to which attention has already been drawn on page 100. A single observation of velocity head, taken with a Pitot-static tube arranged co-axially with the nozzle in the outlet plane, will give a good approximation to the mean velocity of the jet leaving the nozzle, and this velocity, multiplied by the area of the nozzle throat, will give the volume flowing. In cases where the highest accuracy is required, a preliminary exploration of the jet by means of the Pitot-static tube may be desirable in order to verify the constancy of velocity across the jet. A uniform distribution of this nature was only observed in the case of normal smooth pipe distribution of velocity upstream of the nozzle, and it is conceivable that if conditions here are very widely different from the normal, a slight want of uniformity may occur in the jet. For most purposes, however, this refinement will be found to be unnecessary. If the nozzle can be conveniently placed at the outlet end of the pipe a further simplification is possible, since the static pressure in the jet will now be equal to that of the surrounding atmosphere, that is to say, zero, according to the convention by which atmospheric pressure is taken as the datum.* The total head at a point in the jet is therefore equal to the velocity head only, so that all that is required in these circumstances is a single observation of total head at the axis of the jet in the outlet plane of the nozzle. As we have already seen, this total head can be measured by any open-ended tube bent to face the airstream, and no specially shaped tube is now required. A thin-walled tube of about $\frac{1}{16}$-inch bore may conveniently be used. For the reasons mentioned above, a preliminary exploration of the jet may sometimes be advisable in this case also.

**Mean Rate of Flow from a Single Observation.**—A rapid and simple approximate method of determining the mean rate of flow when the distributions of velocity and static pressure at the section of measurement are fairly symmetrical, is to take a single velocity reading at the axis of the pipe. If a sufficient length of straight pipe exists upstream of the section in question, and the pipe is reasonably smooth internally,

* To avoid confusion, the reader is reminded that this convention is followed throughout this chapter.

use may be made of the curve in Fig. 18, obtained by Stanton and Pannell, which shows the relation between the mean and the axial velocities for different values of $\frac{vd}{\nu}$. The axial velocity may be measured either by a small Pitot-static tube, or by means of the small total head tube in conjunction with a static pressure side hole in the pipe as described above. If doubt is felt as to the degree of smoothness of the pipe, or as to whether there is a sufficient length of straight pipe ahead of the section, instead of using the ratio of $\frac{v_{\text{mean}}}{v_{\text{max}}}$ from Fig. 18, the appropriate value of this ratio may be determined once for all by a preliminary careful exploration of the section by means of one of the standard methods already described. If the rates of flow which it is subsequently desired to measure vary over a considerable range, it will be advisable to conduct sufficient preliminary explorations at different rates of flow in order to establish the variation of the ratio $\frac{v_{\text{mean}}}{v_{\text{max}}}$ for the entire range of $\frac{vd}{\nu}$ that will be encountered. Some idea of the variation likely to occur in this ratio can be gained from Fig. 18; it will be seen that for high rates of flow the variation is relatively small, and the amount of preliminary exploration will be governed by the degree of accuracy that is desired. In some cases a single exploration at a rate of flow about half-way between the minimum and maximum likely to be experienced in practice may suffice.

This method may be used for large as well as for small pipes, provided favourable velocity and static pressure distributions exist. It is because a condition making for such distributions, namely, a portion of straight pipe some 20 diameters or so in length, is more usually present in the case of pipes of small diameter that the method finds its largest field of application for such pipes. This condition of a length of pipe upstream of the section of measurement is essential for a symmetrical distribution of velocity and static pressure in a pipe system on the blowing side of a fan, since, in general, all fans deliver the air in a very disturbed turbulent condition, and the flow close to the fan outlet very often does not fill the pipe symmetrically, but is, as it were, piled up to one side or another. On the suction side, however, three or four diameters of pipe upstream of the section may suffice, with a further 6 diameters or so between it and the fan inlet. Although the distribution

of velocity at such a section will naturally not be of the type which we have termed the "normal" * distribution, it will nevertheless usually be symmetrical and have a definite value for the ratio of mean to axial velocity, which can be determined by exploration as already described. A word of caution is necessary for this case. It will probably be found that the open end of the pipe acts to some extent as a sharp-edged orifice, that is, the airstream will contract and not fill the pipe completely for a short distance from the end. This effect can be very largely reduced by fitting the pipe with a shaped inlet, having an easy entry, as shown in Fig. 49 (*a*) and (*b*). The type illustrated in Fig. 49 (*a*) gives better conditions, but is somewhat more expensive to construct; either type may be made in wood and fitted to the pipe.

A modification of this method may often be found con-

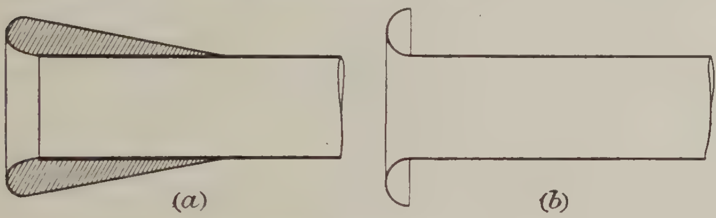


FIG. 49.—Types of shaped inlet to a pipe.

venient. Instead of determining the mean rate of flow from the axial velocity, the total head at the centre of the pipe, or the static pressure at a side hole, may often be utilised. If the flow at the section is parallel, each of these quantities will in many cases be found to vary approximately as the square of the air-speed, so that if P is either the total head or the static pressure at the section, and $v_m$ is the mean rate of flow, we have

$$P = Kv_m^2 \text{ approximately,} \qquad . \qquad . \qquad (1)$$

where K is a constant.

The value of K can be determined experimentally by measuring simultaneously $v_m$, by one of the methods already described, and P for various rates of flow. Once K is established, $v_m$ can be calculated from an observation of P.

It will be useful to examine this method in greater detail. Let us confine our attention to a section X in a given length

* See p. 86.

of pipe AB having a fan at B, which may either be made to blow from B to A or to exhaust from A to B. For simplicity imagine also that there is a shaped entry at A. We shall assume initially that the velocity across the section is uniform and equal to $v_m$. Let O be a point in the undisturbed atmosphere where the air is at rest (Fig. 50).

If we neglect the commencement of the motion when the fan is starting up, and confine our attention to steady conditions when the flow is established and proceeding at a constant rate, we see that the only work done is that against friction, either between the moving air current and the pipe walls or between adjacent particles of air moving at different rates, and the energy equivalent of the internal losses in the current itself due to turbulence. These may be grouped with the frictional losses, since they are inseparable from the flow. We may regard the flow as a complete circuit which commences and ends at the point O in the external atmosphere. The

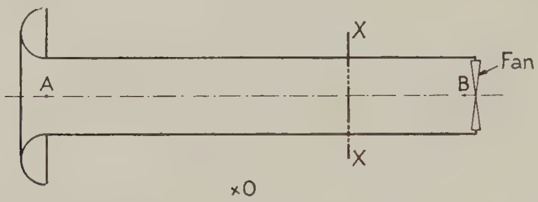



FIG. 50.

air starts from rest, acquires a velocity $v_m$ in the pipe, and is returned to a state of rest. (We shall neglect internal losses in the fan itself.) The change of total head between any two sections of the circuit is equal to the energy loss per unit volume between those two sections.

(*a*) **Fan Sucking Air from A to B.**—Assume that at A, just inside the pipe inlet, the air has acquired its velocity $v_m$.

At O the total head, velocity head, and static pressure are all zero (the atmospheric pressure is taken as the datum according to the usual convention). Between O and A there is a certain loss of head $f_1$, due to friction between moving and still air, and at A the air has acquired its velocity $v_m$. The total head at A therefore differs from that at O only by the amount $f_1$, and is therefore equal to $-f_1$. Bernouilli's equation (which states that, apart from energy losses, the total head is constant), when applied to the portion OA of the circuit, shows that the static pressure at A is $-\frac{1}{2}\rho v_m{}^2 - f_1$, since the velocity head is

$\frac{1}{2}\rho v_m^2$ and the total head $-f_1$. Between A and X there is an additional energy loss $f_2$ due to the frictional resistance of the pipe between A and X,* and the static and total heads at X may again be determined by the application of Bernouilli's equation. We obtain in this manner

$$\text{Total head at X} = -(f_1 + f_2),$$

and

$$\text{Static pressure at X} = -(f_1 + f_2 + \tfrac{1}{2}\rho v_m^2),$$

the negative signs indicating that both these pressures are less than atmospheric.

Now $f_1$ is always found to be very small compared with the other pressures, so that we may say that

$$\left.\begin{aligned}&\text{Total head at X} = -f_2,\\ \text{and}\quad&\\ &\text{Static pressure at X} = -(f_2 + \tfrac{1}{2}\rho v_m^2)\end{aligned}\right\}\text{approximately.}$$

Moreover, if there is a shaped entry at A, the quantity $f_2$ will not differ appreciably from the frictional resistance of the length of pipe between A and X, and this will be related to the mean rate of flow according to equation (4) on page 40. Although this frictional resistance does not vary exactly as $v_m^2$, the error made by assuming that it does so vary will not be large even over a fairly wide range of speed, and for most practical purposes we may therefore take both static pressure and total head at X to vary as the square of the speed, so that equation (1) is established for the case where X is on the suction side of the fan. Usually $f_2$ will be found to be considerably smaller than $\frac{1}{2}\rho v_m^2$, the velocity head. For the rates of flow for which the method of measurement under consideration can be used, that is to say for which the pressures are of sufficient magnitude to be measurable with the necessary degree of accuracy, the section X will have to be something like 50 pipe diameters from A before the frictional resistance of the length AB becomes equal to the velocity head.† As X will usually be considerable nearer than this to A, it follows that at a section on the suction side of a fan the total head is in general small, and the static pressure somewhat greater than the velocity head. Hence

* $f_2$ includes any losses due to the inlet; these are small if there is a shaped entry to the pipe at A.

† This can easily be shown by equating the expression for $p_A - p_B$ in equation (5) on page 40, to $\frac{1}{2}\rho v^2$, and solving for $l/d$.

if this method of measurement is adopted it is preferable, on the suction side, to utilise the static pressure as a measure of the speed, either by adopting an expression such as (1) on page 141, or by plotting a curve between the static pressure and the velocity. It should be noted that, since the portion $f_2$ of the static pressure is usually less than $\frac{1}{2}\rho v_m^2$, the error due to the assumption that the static pressure varies as $v_m^2$ is correspondingly reduced, as it only affects the smaller part $f_2$.

The conclusions to which the reasoning has led will not be invalidated if there is no shaped entry at A; the effect of this will simply be to alter the value of K in equation (1).

(*b*) **Fan Blowing Air from B to A.**—We are again concerned only with the length XA of the pipe. Assume that the mean rate of flow is $v_m$ as before.

At A the air will have a velocity head $\frac{1}{2}\rho v_m^2$ and the static pressure referred to the atmospheric pressure will be zero, so that the total head at A will be $\frac{1}{2}\rho v_m^2$. The resistance of the length AX will of course be $f_2$ as before ($f_2$ does not now, however, contain anything equivalent to the inlet loss of the previous case, and so will have a slightly different value), so that the total head and static pressure at X will both be higher by $f_2$ than they are at A. Thus

$$\text{Static pressure at X} = f_2.$$
$$\text{Total head at X} = f_2 + \tfrac{1}{2}\rho v_m^2.$$

Following a similar line of argument to that previously adopted, we arrive at the conclusion that in this case, i.e. when X is on the delivery side of the fan, the total head, being greater than the static pressure, should be taken as a measure of the speed, and that it varies very nearly as the square of the speed.

In both cases (*a*) and (*b*) the assumption has been made that the velocity $v_m$ is uniform across the section. Actually, of course, it is not, but the effect of this will merely be to alter the value of K in equation (1).

# CHAPTER IX.

## MANOMETERS.

We have already had occasion to note that the velocity heads which have to be measured in practice are usually not large: for example, the velocity head, under ordinary atmospheric conditions of temperature and pressure, does not reach 1 inch of water until the air speed is roughly 66 feet per second. Further, since the head is proportional to the square of the speed, it falls off rapidly at the lower speeds; it is, in fact, less than 0·1 inch of water at 20 feet per second, a speed by no means outside the ordinary working range. It follows that in order to determine low air speeds with sufficient accuracy by means of Pitot and static observations, sensitive manometers, capable of reading pressures of the order of 0·001 inch of water, or even less, will often be necessary.

Numerous and varied devices have been designed to supply this need. A number of these are, for different reasons, unsuitable for industrial use, and it is not proposed in this chapter to attempt a description of all the types of manometers that have been constructed or suggested. We shall confine our attention only to one or two types of different degrees of sensitivity, which have been proved by experience to be suitable for engineering practice, as distinct from laboratory conditions. The main difficulty in designing such manometers is experienced in combining high precision with the portability, robustness, and ease and rapidity of manipulation which are essential for their successful application to general practical use.

**The Tilting Micromanometer.**—This instrument is capable of giving all the accuracy that will be required save in exceptional circumstances. It is, in principle, a sensitive U-tube water gauge in which provision is made for microscopic observation of the water level in one of the vertical limbs. The U-tube is shaped as shown in Fig. 51; it consists of a long horizontal glass tube A, communicating at either end with

the glass vessels BB, whose axes should be parallel and approximately vertical in the zero position. Each vessel is provided with a removable glass cap, ground in to fit, to which rubber tubes leading to the static and Pitot heads may be attached. Short vertical glass tubes with sealed ends project from the base of each cup and fit into corks, which in turn fit firmly

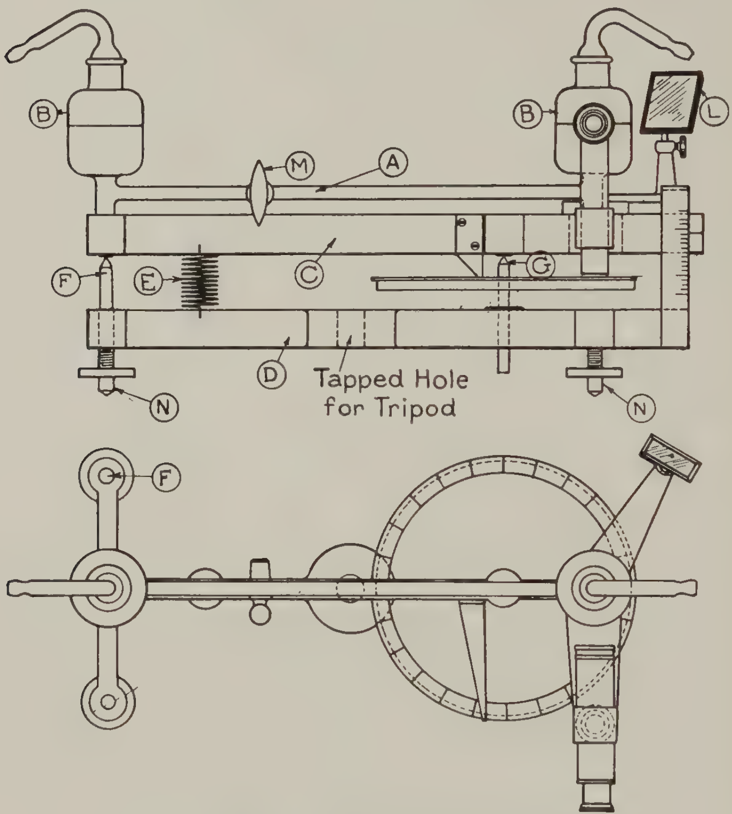



FIG. 51.—Tilting micromanometer.

in holes in the upper movable steel frame C. The latter rests, by means of a three-point support, on the lower fixed frame D, and is held firmly thereon by the spring E. The three-point support is arranged with two points FF under the left-hand vessel, the line joining these points being perpendicular to the axis of the tube A. These two supports are fixed, and the upper

frame, with the U-tube, is tilted about the line joining them by means of the third supporting point G, which is formed by the hardened end of a vertical micrometer screw working in the lower frame. Of the two left-hand points, one engages a conical cup on the upper frame and the other a V-groove with its length horizontal and perpendicular to the axis of the tube A ; the end of the micrometer screw bears against a small flat surface on the upper frame. In this way the tilting frame is given definite location without constraint. All the points and their seatings are of hardened steel. The position of the movable frame with respect to the fixed frame is observed by means of two indicating marks on the former, of which one moves over the graduated micrometer wheel and records fractions of a turn of the screw, and the other indicates complete turns on the fixed vertical scale on the lower frame. A microscope K, with cross-wires in the eyepiece, is carried by the upper frame for observing the level of the liquid in the right-hand vessel, and it is advisable for facilitating observation to have also a small adjustable mirror L, by means of which light from any convenient source can be reflected through the glass vessel into the microscope. The field of vision is then brighter above the water level than below, and the line of demarcation can be sharply focussed. A tap M should be provided to prevent spilling of the water whilst the manometer is being moved from place to place.

The instrument is often best used when mounted on a tripod. For this purpose it is desirable to provide the lower frame with a screwed hole to take a tripod such as that ordinarily used for Dumpy levels or theodolites. It is as well to have also three levelling screws NN on the lower frame to adjust the zero of the instrument if it should be found more convenient to rest it on a bench or table. A further convenient practical step is to bore eccentric holes to take the two cup supports in the two corks fitting into the holes provided in the tilting frame. In this manner some adjustment is provided for small variations from the standard dimensions in the glassware, since it is not possible to make this exactly to size in every case.

In using the manometer, the tubes from the Pitot and static heads should be connected to the cups before the zero reading is taken, as the instrument may easily be displaced whilst the tubes are being fitted. It may be noted here that the caps fitting into the necks of the cups should be greased with a small quantity of vaseline over the region of contact in order

to ensure the absence of leaks at the joints. The glass tube A should initially be approximately horizontal, and the zero reading is obtained by observing the readings on the fixed scale and the micrometer wheel when the cross wires of the microscope intersect the image of the water level. The total head and static tubes are then inserted in the pipe in which the air speed is being measured, and the movable frame is tilted by means of the screw until the image of the water level again coincides with the intersection of the cross wires.

Let $d$ be the difference between the initial and final readings, expressed in turns and fractions of a turn of the wheel ;

$l_1$ be the distance between the cup centres in inches ;

$l_2$ be the perpendicular distance of the axis of the micrometer screw from the line joining the two fixed points ;

$t$ be the pitch of the micrometer screw in inches ;

and $p$ be the pressure difference in inches of water.

Then, if the angle of tilt be not too great,

$$p = td\frac{l_1}{l_2}. \quad . \quad . \quad . \quad . \quad (1)$$

Convenient practical values of $l_1$, $l_2$, and $t$ are 13 inches, 10 inches, and 0·05 inch respectively. The available travel of the micrometer screw should not exceed 20 turns, giving the instrument a range of about $1\frac{1}{4}$ inches of water, and the diameter of the screw over the thread should be at least $\frac{3}{8}$ inch. Smaller sizes lead to " sloppiness " as the thread wears.

If a higher range is required, it should not be obtained by increasing the length of the screw, since the large angles of tilt which this course entails at the higher end of the range introduce errors for which it is necessary to make due allowance, and the above equation is no longer valid. It is preferable to increase the distance between the cup centres ; a 26-inch U-tube used in place of a 13-inch tube would have double the range, and such a tube could quite conveniently be mounted on a frame designed for a 13-inch tube. It will be found, however, in practice, that when the velocity head exceeds that covered by the range of a 13-inch tilting manometer, the use of an ordinary U-tube water gauge will probably give sufficient accuracy in most cases.

An important consideration in connection with the tilting manometer is that it is an absolute standard which does not

require calibration against another manometer. Of the dimensions which determine the reading for a given applied pressure difference, two, viz. the pitch of the screw thread and the distance of its axis from the line joining the fixed points, can be made to size sufficiently closely to limit errors in the readings to less than 0·1 per cent. The third important dimension, that is, the distance between the cup centres, cannot be made to a given size with such precision, but can be measured by calipering to 0·01 inch without difficulty. With careful workmanship the overall accuracy of the instrument should be within 0·1 per cent. over the entire range.

If this degree of accuracy is required at the higher pressures, a small correction must be applied to the readings in this part of the range. In equation (1) the product *td* represents the height *h* in inches through which the micrometer screw moves, that is the vertical distance by which one point on the movable frame is raised or lowered above another point, which does not move vertically. The assumption underlying equation (1) is that the length of the line joining these points is constant and equal to $l_2$, the distance between the axis of the micrometer screw and the fixed axis of rotation of the tilting frame. Actually, of course, the length of this line on the moving frame, i.e. the "leverage," is not constant, and only has its value assumed in equation (1) in the initial position. As the micrometer screw moves, its point slides along the hardened steel surface on the moving frame with which it is in contact, so that the leverage changes. It is easy to show that the true leverage for the reading *d* in equation (1) is $\sqrt{l_2^2 + t^2d^2}$ instead of $l_2$ as is therein assumed, so that the true pressure difference is obtained from the equation,

$$p = td\frac{l_1}{\sqrt{l_2^2 + t^2d^2}}. \qquad . \quad . \quad . \quad (2)$$

The error incurred by using (1) instead of (2) is only 1 per cent. when $td = 1·4$ inch, and decreases as *td* becomes smaller relatively to $l_2$.

The necessity for applying this correction can be entirely avoided if the upper end of the micrometer screw terminates in a horizontal, hardened, flat surface instead of in a point, and a hardened steel ball, fixed to the moving frame and resting on the flat end of the micrometer screw, is substituted for the plane bearing surface. The leverage will then be constant and

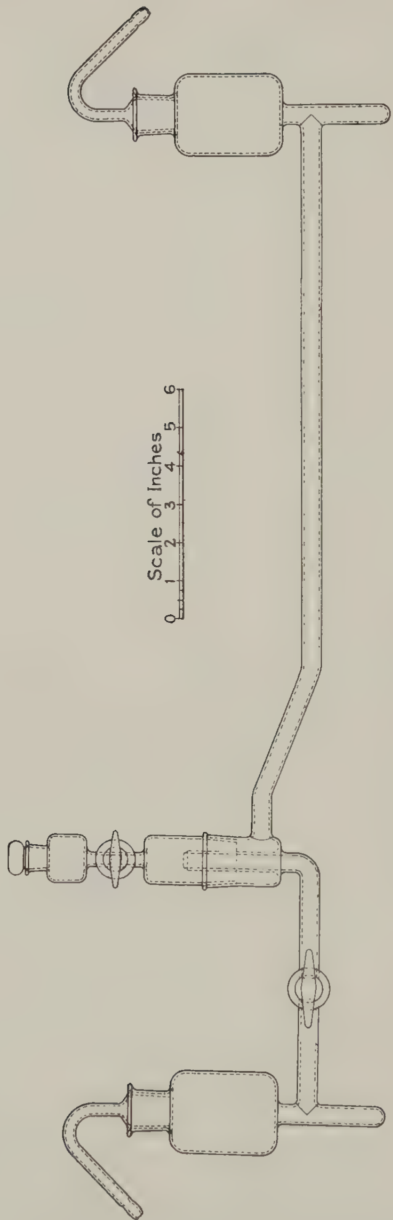



FIG. 52.—Glass-work for 26-inch Chattock manometer.

equal to $l_2$, the distance between the line joining the two fixed left-hand points (FF in Fig. 51) and the centre of the steel ball. The latter will now simply slide on the fixed end of the micrometer screw.

With a microscope having a magnification of 20 or 30 diameters the tilting manometer as described will be sensitive to pressures of less than 0·001 inch of water, which means that when this instrument is used with an ordinary Pitot-static tube a wind speed of 10 feet per second can be observed with an accuracy of about 2 per cent. By a modification of the glass-work the sensitivity can be greatly increased, unfortunately, however, at the expense of ease of manipulation and rapidity of working. The alteration consists in cutting the horizontal limb of the U-tube and connecting the two parts to a central vessel, one part being connected to the outside of this vessel near its lower end and the other being bent vertically upwards and passing up into the centre of the vessel. The modified arrangement can be seen from Fig. 52 which illustrates the glass-work of a manometer constructed on these

lines, and from Fig. 53 which shows a complete instrument. It will be seen that the metal frame is of the type previously described.

This design of glass-work was first suggested by Chattock.* The vertical tube in the central vessel extends upwards for some distance as shown and is cut off square and ground. Distilled water is poured into the lower parts of the two cups and the central vessel, and into the upper portion of the latter, completely filling it, is admitted a quantity of medicated paraffin ("Nujol"), by the aid of the filling reservoir and tap shown in the diagrams. The paraffin does not mix with water, and

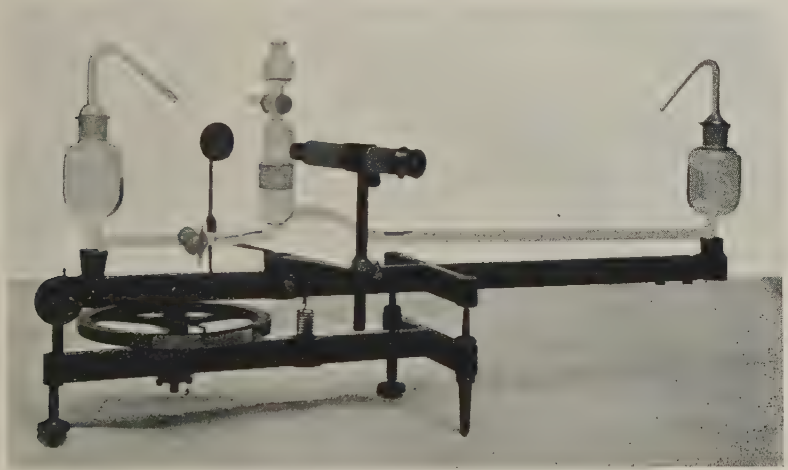


FIG. 53.—Chattock manometer (26-inch cup centres).

two well-marked surfaces of separation are formed, one outside the vertical tube in the interior of the central vessel and one within this tube. The liquid levels are adjusted, by regulating the quantities admitted, so that one surface of separation is formed on the open end of the central vertical tube, and has the appearance of a bubble resting on the mouth of the tube. If now a pressure difference is applied to the two end cups this bubble tends to become larger or smaller and its movement, which can be followed by suitable illumination from behind and microscopic observation, is arrested by giving an appropriate tilt to the frame by means of the micrometer screw.

* *Phil. Mag.*, 1901, p. 83.

Instruments of this type, having cups whose centres are 13 inches apart, are sensitive to pressure differences of the order of 0·0001 inch of water, the increased sensitivity being attributable to the fact that the motion of the liquid is observed in a narrow tube instead of in the relatively wide end cups. Subject to neglect of surface tension effects, the motion of the bubble for a given pressure difference will be greater than the motion of the surface in one of the end cups in the ratio, approximately, of the area of the water surface in the cup to the internal area of the central tube. With a microscope of given magnification, therefore, the sensitivity of a Chattock type of manometer will be greater in this ratio than that of an ordinary tilting manometer. Actually, surface tension effects are by no means negligible, and reduce the sensitivity considerably. Nevertheless, as will be seen from a comparison of the figures quoted, this instrument is considerably more sensitive than the plain tilting manometer. Its chief disadvantage from an engineering standpoint is that, particularly with fluctuating pressures, the surface of separation is easily broken, in which case measurements have to be interrupted whilst a fresh zero setting is taken. Further, small zero changes may also occur as the temperature alters, on account of the different coefficients of thermal expansion of the water and the paraffin. A modification to eliminate this zero creep has recently been introduced by W. J. Duncan,* and for further details of the Chattock gauge the reader is referred to this paper and an earlier one which appeared in *Engineering* in 1913.†

**Inclined Tube Manometers.**—If a small sacrifice of accuracy, and the advantage attendant upon the use of an instrument which does not require calibration, are considered unimportant, a convenient form of gauge suitable for factory use will be found in the Krell manometer, shown diagrammatically in Fig. 54. The principle of this instrument is simple: if one of the arms of a U-tube is inclined at a small angle to the horizontal, the liquid in that arm will move a considerable distance along the tube for a small amount of vertical motion, i.e. for a small applied pressure difference. Reference to the diagram shows that the gauge consists essentially of an inclined tube A, which forms one arm of the U-tube, and a large reser-

* *Journal of Scientific Instruments*, vol. iv., 1927, p. 376.

† "The Chattock Tilting Manometer," J. R. Pannell, *Engineering*, 12th Sept., 1913, p. 343.

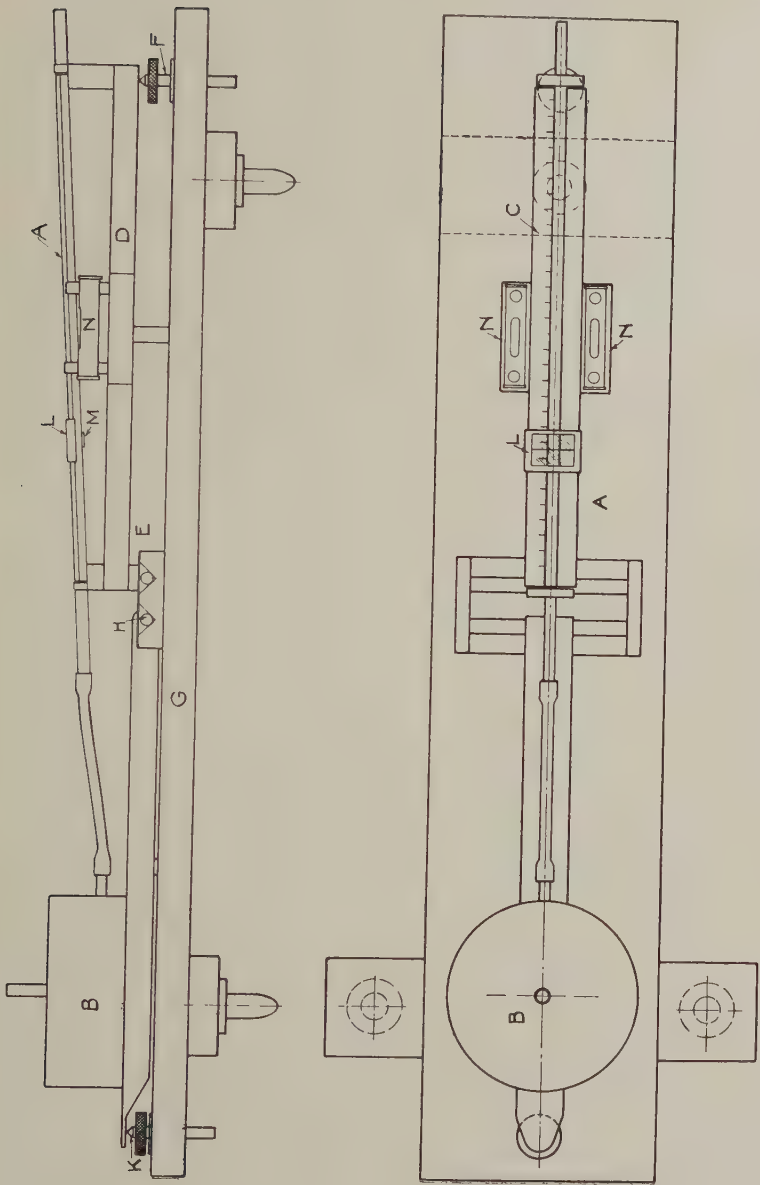



FIG. 54.—Krell type inclined tube manometer.

voir B, having an area several hundred times that of the tube, forming the other arm. The sources of pressure are connected to the reservoir and to the end of the inclined tube ; the use of a large reservoir causes the liquid level therein to remain virtually stationary, practically all the motion occurring in the tube, where the position of the meniscus is observed against a graduated scale C.

The tube is held on the frame D, which is pivoted at E, and can be raised or lowered at the other end by means of the screw F, so that the inclination of the tube can be altered and the sensitivity and range thereby varied. The whole instrument is mounted on a baseboard G, fitted with three levelling screws, and provision is made for rotating the reservoir support about the fulcrum H by means of the screw K, in order to adjust the zero level of the liquid in the inclined tube. For observing the position of the meniscus it is convenient to have a sliding frame L, which moves along the scale and carries a cross wire above the tube, and a small strip of white ivorine M below.

It is usual to provide two spirit levels NN, which are adjusted and locked when the manometer is initially calibrated so that the tube may be set to two definite inclinations. The instrument then has two ranges for indicating low and high pressures respectively. The advantage of this will be realised when it is remembered that the velocity head is proportional to the square of the wind speed, and that it is usually necessary to allow for the measurement of fairly high speeds. If, therefore, the tube is set to such an inclination that the full scale reading corresponds with the highest wind speed liable to be encountered, the readings for low wind speeds will be small, and very little accuracy will be obtained in this range.

Alcohol is found to be the liquid giving the best meniscus in this type of manometer. As regards the bore of the inclined tube, a diameter of about 0·2 inch gives the best results ; * a convenient length of tube is one with which a full scale reading of 12 inches can be obtained. With such a tube set at an inclination of 11 degrees, the highest pressure indicated, if alcohol is used as the manometric liquid, will correspond with a wind speed of about 80 feet per second. If this is adopted as the higher range, a convenient lower range will probably be found to be that in which the second level is set at $2\frac{1}{2}°$, when the maximum pressure will correspond with an air speed of about

* Hunsaker, *Smithsonian Misc. Coll.*, vol. 62, p. 34.

40 feet per second. The scale can be read to about 0·02 inch, and an accuracy of 3 per cent. or so will be obtained at a pressure corresponding with 10 feet per second on the lower range. If this degree of accuracy at low speeds is not considered sufficient, the levels can be adjusted to smaller angles ; at higher speeds the precision is, of course, much better.

The manometer is not a fundamental standard, and requires calibration against a primary manometer, since various factors, such as lack of straightness of the inclined tube and surface tension effects, due to want of uniformity in the bore, preclude the possibility of calculating true pressures sufficiently accurately from the observed readings and the known inclination of the tube. In order, however, to estimate initially the angle to which the tube should be set for any desired range, the following formula will give sufficiently exact results. It is the formula which would be valid if the instrument were not subject to the errors mentioned above :—

$$\text{Sin}\ \theta = \frac{h}{l\sigma}, \qquad . \quad . \quad . \quad . \quad (3)$$

where $h$ is the maximum water column to be observed,
$l$ is the total length of the scale,
$\sigma$ is the specific gravity of the liquid,
and $\theta$ is the required slope of the tube to the horizontal.

Manometers of this type are to be recommended for general use : when calibrated they are accurate and consistent in their readings and comparatively inexpensive when their degree of performance is taken into consideration. They are, moreover, convenient and robust, and by their aid observations of pressure can be rapidly performed. The only disadvantage is the fact that each manometer requires calibration initially against a standard instrument. A certificate giving the results of calibration can be obtained by submitting the instrument to the National Physical Laboratory for test ; in many cases the manufacturers will obtain an N.P.L. certificate and supply it with the gauge. If an absolute standard, such as the tilting micromanometer, is available, the calibration can easily be carried out in the manner described at the end of this chapter.

A modification of the inclined tube manometer has been designed by Hodgson, in which the manometer tube, instead of being inclined at a constant angle along its entire length, is curved in such a manner that a uniform scale of velocity is obtained. This instrument is particularly useful when,

as is often the case, it is desired to have an instrument continuously installed to indicate instantaneously the speed or quantity of air flowing in a pipe line. A form of this manometer fitted with a pressure scale is shown in Fig. 55, which is reproduced by the courtesy of Messrs. George Kent, Ltd., Luton. With a velocity scale the instrument can be used, together with the Pitot and static tube, to indicate speed in any pipe, but with a quantity scale it can, of course, only be used for the particular pipe for which it is designed.

Instruments of this type, in addition to the advantage they offer in enabling wind speeds to be rapidly determined, have the merit that a wide range of speeds can be observed with one tube at a constant setting. On the other hand, they

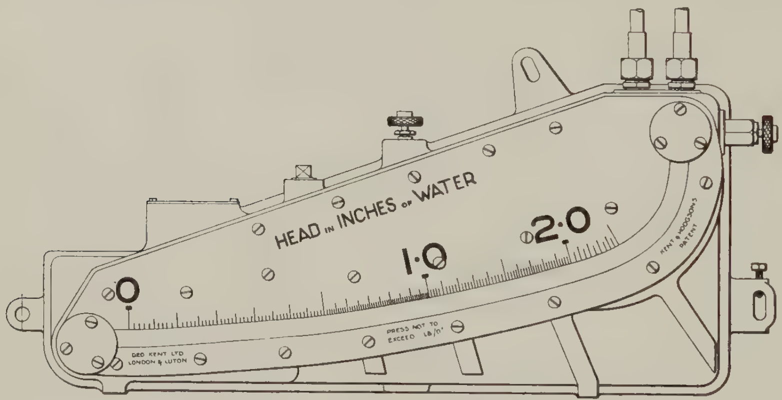



FIG. 55.—Curved tube manometer.

are probably not so convenient for general use as instruments graduated directly in terms of pressure, since they can only be employed to measure pressures by fitting them with a different tube and scale. It must also be remembered that a particular velocity (or quantity) scale is only accurate at one particular value of the air density ; as this property varies with temperature and pressure, corrections have to be applied to the readings. Normally, however, these corrections will be quite small. Attention is drawn to this point in the discussion of the theory of the instrument given at the end of the present chapter.

**Large Range Micromanometers.**—A useful and robust type of manometer is that employing virtually a flexible U-tube, consisting of two limbs whose lower ends are connected by a length of rubber tubing One of the limbs remains stationary

whilst the other is raised or lowered, by an amount corresponding to the applied pressure difference, by means of a micrometer screw to which it is attached. Fig. 56 illustrates a manometer which utilises this principle and incorporates some useful features introduced at the University of Toronto.* The construction of the instrument is clear from the figure, and needs

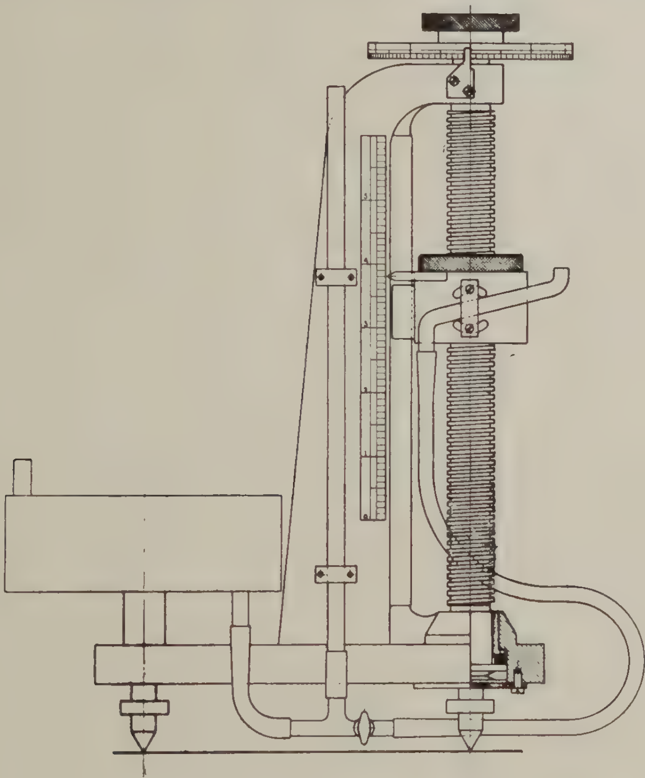


FIG. 56.—Large range micromanometer.

little comment. Rotation of the nut supporting the movable limb of the U-tube is prevented by means of a keyway, which engages with a vertical key cut on the right of the vertical supporting standard carrying the head of the micrometer screw. In place of a cup for the moving limb, an inclined glass tube

* *School of Engineering Research*, Bulletin No. 2, Paper No. 1, 1921 (J. H. Parkin).

of variable slope is employed having a line etched on it. The zero reading is first established with the liquid meniscus in this tube in coincidence with the fixed mark. When a pressure difference is applied the inclined tube is raised on its nut until the liquid is again in equilibrium at the mark. Complete turns of the micrometer screw are indicated by the pointer moving over the vertical scale, whilst fractions of a turn are observed on the graduated head. The fixed limb of the U-tube consists of a reservoir of large cross-sectional area in comparison with that of the inclined tube, so that the change in level in the reservoir can be neglected. A convenient feature of this manometer is the vertical glass tube fixed alongside the scale as shown. This tube is connected to the reservoir in parallel with the inclined tube by means of a T-piece, and if its open end is connected to the same source of pressure as the open end of the inclined tube, the level of the liquid in it indicates the approximate height to which the nut should be raised when the order of magnitude of the applied pressure difference is unknown beforehand. The final adjustment is made by the aid of the meniscus in the inclined tube.

As in all types of manometer in which the meniscus in an inclined tube is observed, alcohol is found to be the most suitable liquid, and the best sensitivity for general use appears to be obtained by the use of an inclined tube of about 0·2-inch bore set at a slope of 3° or so, corresponding to a magnification of roughly 20 to 1. With smaller slopes no appreciable increase of sensitivity can be gained without making the instrument unduly sluggish. Tubes of smaller bore reduce the sluggishness to some extent, but the meniscus is not so good.

This instrument, though not as simple or rapid in use as the plain inclined tube manometer, is certainly more accurate, and has the advantage that it does not require calibration; the readings need correction only for the specific gravity of the alcohol used, in order to convert them into inches of water. Once the accuracy of the micrometer screw thread has been checked the manometer can therefore be used as an absolute standard. It is stated * that the movement of the liquid column can be read to within about 0·0002 inch on the average, with an extreme possible error of 0·0005 inch. A further important advantage of this instrument is the fact that the range is not as restricted as it is in other sensitive manometers, since the micrometer screw may be made 10 or 12 inches long.

* Parkin, *loc. cit.*

A manometer similar in principle and construction to that just described has been designed for use at the National Physical Laboratory.* In this instrument the equilibrium position is determined in a different manner. The two limbs of the U-tube in this case consist of two glass cups similar to those used in the tilting manometer. One of these cups is raised or lowered by means of the micrometer screw, whilst the other is fixed. The latter cup is constructed as shown in Fig. 57; it will be seen that the arrangement is precisely similar to that of the central vessel in the Chattock manometer, and, as in that instrument, use is made of a water-paraffin meniscus on the mouth of the central tube to indicate the equilibrium position. The meniscus is suitably illuminated and is viewed through a microscope. For the zero setting the meniscus is adjusted so that its image coincides with the intersection of the cross wires in the eyepiece of the microscope; an applied pressure difference is then balanced by adjusting the level of the movable cup until the image of the bubble once more coincides with the intersection of the cross wires. In this instrument, the cross-sectional area of the rubber connecting tube is not small compared with that of the cups. Hence in order to avoid appreciable errors due to change in volume of the rubber tubing, it is advisable to have a long length of tubing so that it hangs approximately vertically from each cup over the entire range of the movable cup. The changes of shape that then occur in the tubing will only produce negligibly small volume changes.

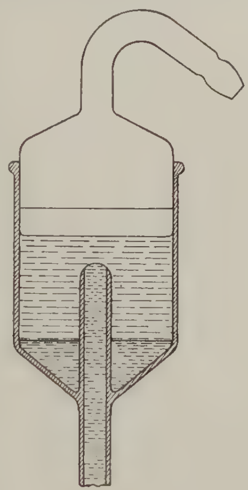


FIG. 57.

**The Two-Liquid Differential Manometer.**—A convenient instrument for work where high accuracy is not required is shown in Fig. 58. It is, in reality, a modified form of U-tube, magnification of the readings being secured by the use of two non-miscible and mutually insoluble liquids of nearly equal densities. A serious fault of the plain U-tube—namely, zero errors due to tilt of the instrument—is overcome in the manometer under consideration by arranging the two limbs

* N.P.L. Report for 1921, p. 170.

concentrically as shown.* Each limb is surmounted by a reservoir, connection to the reservoir of the inner tube being made by means of a short glass tube passing through the top of the outer reservoir, to which it is sealed on the outside as shown. The two liquids are arranged with the lighter one in the inner tube, the surface of separation or meniscus being near the upper end, and pressure readings are obtained in terms of the movement of the meniscus down the central tube. The pressure difference should be applied so that the higher positive pressure acts on the inner reservoir, thus giving a downward movement to the meniscus.

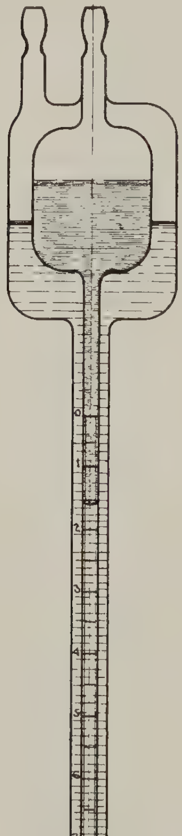


FIG. 58. — Two-liquid differential manometer (concentric tubes).

The relation connecting the applied pressure difference in inches of water with the movement of the meniscus can easily be deduced as follows :—

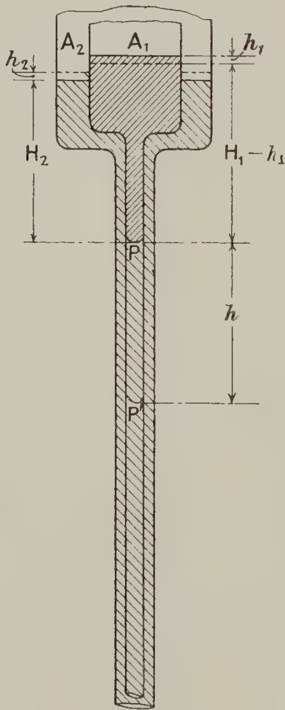



FIG. 59.

In Fig. 59 let $A_1$ and $A_2$ be the areas of the inner and outer reservoirs respectively, and $a$ the

* This type of two-liquid manometer was originally designed at the Royal Aircraft Establishment for use on aeroplanes, see Reports and Memoranda of the Advisory Committee for Aeronautics, No. 144. The instrument mentioned in this Report had the lighter liquid in the outer reservoir, the surface of separation of the two liquids being in the outer tube. The form here described is preferable, since a more definite meniscus is obtained.

area of the central tube. Also, in the zero equilibrium position before a pressure difference is applied, let the boundary between the lighter and heavier liquids be at a point P in the central tube such that the free levels in the two reservoirs are at heights $H_1$ and $H_2$ above P as shown. Also let $\sigma_1$ and $\sigma_2$ be the specific gravities of the lighter and heavier liquids respectively.

From a consideration of the equilibrium of the liquids in the inner and outer tubes at the level of the point P, we have the equation,

$$H_1\sigma_1 = H_2\sigma_2. \qquad (4)$$

Now let a pressure $p$ inches of water be applied to the inner reservoir. The meniscus will move down a distance $h$ to the point P′ and at the same time the level in the inner reservoir will fall and that in the outer reservoir rise, by amounts $h_1$ and $h_2$ respectively.

The equilibrium condition for the level P′ now becomes

$$p + (H_1 - h_1 + h)\sigma_1 = (H_2 + h_2 + h)\sigma_2. \qquad (5)$$

The volume of liquid displaced in each reservoir must be equal to the volume of liquid displaced in the inner tube. That is

$$ah = A_1h_1 = A_2h_2.$$

Hence we may write

$$\left.\begin{aligned} h_1 &= \frac{a}{A_1}h, \\ h_2 &= \frac{a}{A_2}h. \end{aligned}\right\} \qquad (6)$$

Substituting the relationships of equations (4) and (6) in (5), we obtain

$$p_1 + h\sigma_1\left(1 - \frac{a}{A_1}\right) = h\sigma_2\left(1 + \frac{a}{A_2}\right),$$

or

$$p = h\left[(\sigma_2 - \sigma_1) + \frac{a(\sigma_1A_1 + \sigma_2A_2)}{A_1A_2}\right], \qquad (7)$$

which is the relation between the applied pressure difference $p$ in inches of water and the corresponding displacement $h$ of the meniscus.

The magnification is obviously equal to the reciprocal of the term in square brackets on the right-hand side of (7), and from this it is evident that, for a manometer of given dimensions, the more nearly equal the specific gravities of the two liquids used, the greater will be the magnification. Unfortunately, in practice the magnification that can be usefully employed cannot be increased indefinitely without introducing serious disadvantages and causes of inaccuracy. If $\sigma_1$ and $\sigma_2$ differ only by a small amount (5 per cent. or less) the instrument must have long tubes in order to give readings over a reasonable range of pressure, and the movement will be sluggish. Moreover, when the densities are too nearly equal, trouble is experienced with the meniscus, which becomes irregular in form and tends to break up. It is not advisable to aim at a magnification greater than 10 or 12 to 1, under which conditions the gauge should prove fairly satisfactory. A further factor which sensibly influences the performance of the instrument is the bore of the tubing, which should not be so large that the shape of the meniscus is under insufficient capillary control, nor so small that the capillarity forces are excessive, in which case the liquid column will break if the pressure difference applied to the instrument is subject to rapid changes or fluctuations.

The choice of suitable liquids is also a matter of some importance. As stated above, the primary requirements are that they should neither mix nor be soluble in one another. It is desirable also that the meniscus should be of definite shape and that the liquids should not adhere strongly to the walls of the tube when the surface of separation moves. Since, moreover, the indications of the instrument are dependent upon the densities of the two liquids, the latter should as far as possible be chosen with low and approximately equal coefficients of thermal expansion, so that the calibration does not alter appreciably with temperature fluctuations. Finally, volatile liquids should preferably not be employed, since their use leads to progressive zero changes as evaporation of one or both of the liquids occurs.

Various liquids have been used in gauges of this type, but none has been found to give entire satisfaction. The author has obtained good results with a gauge having a magnification of 10 to 1 in which benzyl alcohol and an aqueous solution of calcium chloride were used, the former liquid being coloured by the addition of a small quantity of aniline black,

soluble in oil, in order to throw into greater prominence the surface of separation of the two otherwise colourless liquids. A number of instruments of this type using these liquids are in use and are proving fairly satisfactory. The advantage of using a liquid such as calcium chloride solution is that its specific gravity can be varied by the amount of the solid salt added, so that when used in conjunction with a liquid of definite specific gravity (in this case benzyl alcohol) different degrees of magnification can be secured. In the gauge under consideration, the specific gravity of the alcohol was 1·048, and that of the heavier calcium chloride solution was adjusted to give a magnification of 10. The variation in the calibration of the instrument with temperature was found to be about 3 per cent. for a temperature range of 12° C., the magnification being higher by this amount at the lower temperature. Errors due to this cause can to a large extent be eliminated by immersing the gauge in an outer vessel containing water, whose temperature is always adjusted to a convenient constant value (probably 65° F.) before readings are taken. An alternative and perhaps easier course is to apply corrections determined by calibrating the gauge at different temperatures. Since the coefficients of thermal expansion of the liquids will be, to a first order, linear, it is sufficiently accurate to assume a linear variation for calibration with temperature. Adequate information for the determination of the correction factor for any temperature will, therefore, in general be obtained by calibrating the gauge at two temperatures, which should preferably be chosen at the extreme ends of the range to which the instrument is likely to be subjected.

Before the liquids are introduced into the manometer it is advisable to shake them well up together in a large flask and to allow them to separate out. Each should then be filtered in order to remove small bubbles of the other and poured separately into its appropriate reservoir. In this manner each of the two liquids is saturated with the other before being introduced into the gauge glass, so that no subsequent changes of density (and hence of calibration) are likely to occur from diffusion and interchanges of water content at the common surface. In addition to this precaution it is essential—and the importance of this point cannot be overemphasised—for the glass-work to be thoroughly chemically clean before the liquids are introduced. The slightest trace of grease on the interior of the inner tube will spoil the definition

of the meniscus, and cause difficulty in reading its exact height. The improvement in the working of the manometer and in the appearance of the meniscus after a dirty instrument has been thoroughly cleaned is remarkable. Cleaning is best effected by means of strong nitric acid and alcohol, or strong sulphuric acid and potassium bichromate.* The reagents should then be removed by washing with distilled water and pure alcohol; the final drying may be accelerated by blowing warm air through the glass-work.

The following internal dimensions are recommended for the various elements of a gauge of this type :—

| | |
|---|---|
| Diameter of outer reservoir . . | 3·5 inches. |
| Diameter of inner reservoir . . | 2·5 ,, |
| Diameter of outer tube . . . | 0·5 inch. |
| Diameter of central tube . . . | 0·2 ,, |

The length of the tubes will naturally be determined by the range of pressure that the gauge is intended to cover. With a magnification of 10, a length of about 24 inches should be sufficient, corresponding with a maximum pressure reading of about 2·3 inches of water, allowing for the necessity of obtaining a zero reading slightly below the top of the inner tube. For pressures greater than this, sufficient accuracy will probably usually be obtained by the use of an ordinary U-tube water manometer. For strength, it is advisable to weld the bottom of the inner tube to the outer tube, and to make provision for the flow of liquid by piercing about four holes in the walls of the inner tube at its lower end. The total area of the holes should be equal to the area of the inner tube. A scale of inches and tenths, or preferably of millimetres, should be engraved on the inner tube for observing the position of the meniscus.

In a more simply constructed but somewhat less reliable form of this gauge, illustrated in Fig. 60, the two reservoirs and the tubes leading from them are arranged side by side instead of concentrically. In this case care must be taken not to tilt the instrument whilst observations are in progress, as appreciable movements of the meniscus will be caused by comparatively small inclinations of the instrument. The for-

* Nitric acid and alcohol are probably the best cleansing agents, but they must be applied cautiously to avoid damage to the glass-work due to too vigorous reaction.

mulæ derived for the type with concentric tubes apply, without alteration, to this gauge.

**Other Types of Manometers.**—From the gauges described above it will be possible to select, at a moderate cost, an instrument suitable to most of the conditions that will have to be met in practice. It must not, however, be thought that the types mentioned represent all the instruments worthy of consideration. For further information reference may with advantage be made to a paper by Hodgson * which contains much interesting and useful matter.

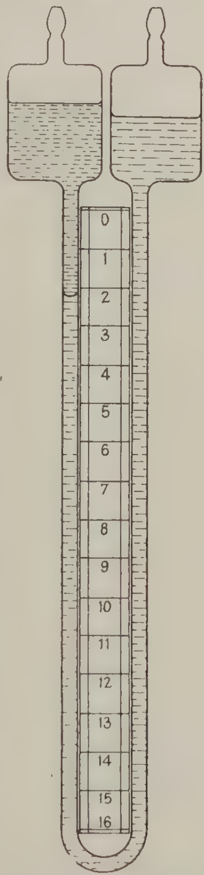



FIG. 60.—Two-liquid differential manometer (parallel tubes).

Before leaving the subject, however, we may consider briefly two further types of sensitive manometers. The first of these, which is due to Hodgson,† is an inclined tube instrument, having a large reservoir fitted with a cylindrical displacer, partly immersed and partly above the liquid in the reservoir, and actuated by means of a vertical micrometer screw. The liquid meniscus in the inclined tube is maintained at a fixed mark, any departure therefrom, due to the applied pressure difference, being compensated by an appropriate vertical motion of the displacer, which alters the liquid level in the reservoir. The meniscus is specially illuminated and is viewed through a microscope having a magnification of 60. The calibration is determined by the relation between the internal diameter of the reservoir and the external diameter of the displacer, so that the instrument may be used as a fundamental standard. The manometer is relatively expensive, but it is robust and portable, and the sensitivity claimed for it is within 0·00002 inch of water. This high precision is attained by the use of a displacer having a small diameter in relation to that of the reservoir.

* "The Measurement of Pressure," *Proc. Inst. Marine Engineers*, vol. 36, Sept., 1924.

† Described in a paper by D. Hay and W. E. Cooke read before the Institution of Mining Engineers on 19th March, 1926.

The second type of sensitive gauge mentioned above is a modification of the Chattock manometer, recently used for a special investigation at the National Physical Laboratory.* Its special feature is the rotation of the glass-work about a vertical axis until the vertical plane through the axes of the cups makes a small angle—10° or so—with the axis of tilt of the frame. In the normal gauge this angle is 90°. This has the same effect as using a gauge-glass having the cup centres considerably closer together than the usual distances of 13 inches or 26 inches. The sensitivity is increased in proportion, but in order to make full use of this improvement, a high-powered microscope is necessary. It is also advisable to use a gauge-glass fitted with Duncan's temperature compensating vessel.

**Calibration of Manometers.**—In calibrating any type of manometer against a standard, it is convenient to take simultaneous readings of the two instruments when one side of each is connected to the same source of pressure, which can be varied at will, the other sides being open to atmosphere or to any constant pressure. For the standard, it is necessary to use an instrument such as the tilting micromanometer, from whose indications, which depend solely upon its mechanical construction and dimensions, the applied pressure differences can be accurately calculated.

The simplest method of conducting the calibration is to open one side of each gauge to atmosphere, and to connect the other sides together through a T-piece and tap by means of which the pressure can be applied to both gauges simultaneously. This method is subject to three serious disadvantages. In the first place, when the tap on the variable pressure side is shut the connecting tubes will enclose a small volume of air, and the small local atmospheric temperature fluctuations which are continually occurring will produce serious changes in the readings of both gauges, so that simultaneous observations of both under steady conditions will not be possible. A similar effect arises from the alteration in the enclosed volume while the standard instrument is being adjusted. This volume should be sufficiently large to render negligible the pressure changes to which it is subjected by the movement of either gauge. Finally, when very small pressure differences are being measured the atmosphere cannot be considered as a source of constant pressure. The small local

* Simmons and Jones, *Reports and Memoranda No.* 1103 *of the Aeronautical Research Committee.*

variations of pressure due to draughts and similar causes would make the attainment of steady conditions impossible if this crude method of calibration were adopted.

A logical extension of the above method in the light of these considerations leads to the system of connections for manometer calibrations shown in Fig. 61, by means of which a sufficient degree of steadiness is secured. Corresponding sides of both gauges are connected in parallel, as shown, to two reservoirs which are completely immersed in a water bath to reduce temperature variation effects. One of these reservoirs, A, forms the constant pressure side, and has no other connections; the pressure in the other, B, can be varied through a second tube leading into it. It is advisable, if possible, to arrange the gauges so that a movement in the positive direction is obtained by decreasing the pressure in B below atmospheric.

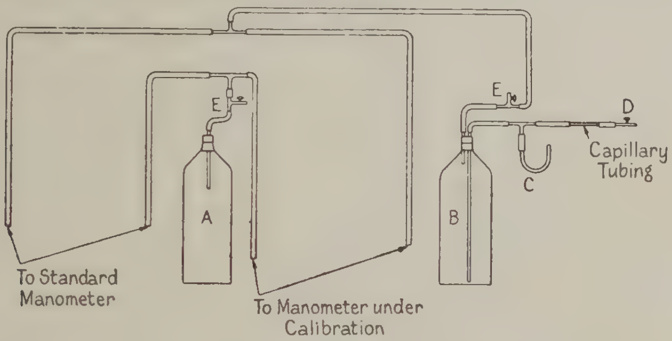



FIG. 61.—System for calibration of manometers.

The pressure can then conveniently be varied by sucking some of the air out of the reservoir; in this way the enclosed air is not warmed as it would be if the pressure were raised by blowing. In order to avoid sudden large variations of pressure while air is being sucked out of B and to provide a means of controlling the applied pressure difference with some degree of precision, it is advisable to have a length of about 4 inches of capillary tubing in the circuit. A further useful addition is the U-tube water manometer C, from which the approximate pressure in B can be observed, so that the tap D can be closed when the desired pressure difference is obtained. It is convenient also to provide bye-passes EE and taps, so that the two reservoirs and both sides of the gauges can be opened to atmosphere to enable zero readings to be taken when required.

For the reasons stated above, the two reservoirs must be of considerable volume: quart Winchester flasks are about the minimum size that should be employed. Some difficulty is usually experienced in making a flask of this type leak-tight. The writer has found that an effective method is to insert the cork or rubber bung with the necessary glass connecting tubes, and then to tie a cylinder of stout paper tightly round the neck, and to pour in sufficient molten paraffin wax to cover the bung and holes through which the glass tubes enter. When solid the wax forms an efficient seal.

By means of this arrangement, calibrations can be rapidly and accurately performed, preferably by two observers taking simultaneous observations on the two gauges.

**Testing for Leaks.**—In all air-flow measurements involving observations of pressure, care should be taken to ensure the absence of leaks in the various instruments, connections, and lengths of rubber tubing employed. Glass joints, such as taps and the caps of the two vessels of the tilting micrometer, should be greased with a thin coating of vaseline, and all parts of the pressure circuit should be tested for leaks. Particular attention should be paid to the concentric type Pitot and static tube, which is liable to develop internal leaks from one tube to the other, and to all rubber tubing, which is apt to perish and crack, especially at the ends where it is stretched over metal or glass connecting pieces. Even new rubber tubing should not be regarded as above suspicion. The author has found appreciable porosity in rubber tubing taken directly from stock. It is advisable therefore to test separately each length of rubber tube used.

A simple leak tester is shown in Fig. 62. It consists of a small U-tube water gauge with two connecting tubes to one of its limbs. One end of the tube or connection to be tested is connected to the tube A by means of a piece of rubber tubing, which has previously been ascertained to be itself free from leaks: the other end is closed, and a small pressure (about $1\frac{1}{2}$ inches to 2 inches of water) is applied by blowing gently through the tube B and closing the tap before the pressure is released. Any small leaks will now be indicated by a gradual creep of the water levels in the two limbs. In order better to observe the motion, it is convenient to have a number of horizontal lines about $\frac{1}{2}$ inch apart ruled on the board to which the U-tube is fixed. The function of the two bulbs in the leak tester is to reduce the risk of ejecting water from the limbs if excessive pressure is inadvertently applied.

In testing concentric Pitot and static tubes for leaks, each tube should be dealt with separately: no difficulty will be found in the case of the impact tube, since its end can be effectively closed by pressing the ball of the thumb against it. In order to test the static tube, the static holes are best closed by winding tightly round the portion of the tube in which the holes are situated, three or four thicknesses of rubber tape as used for electrical insulation.

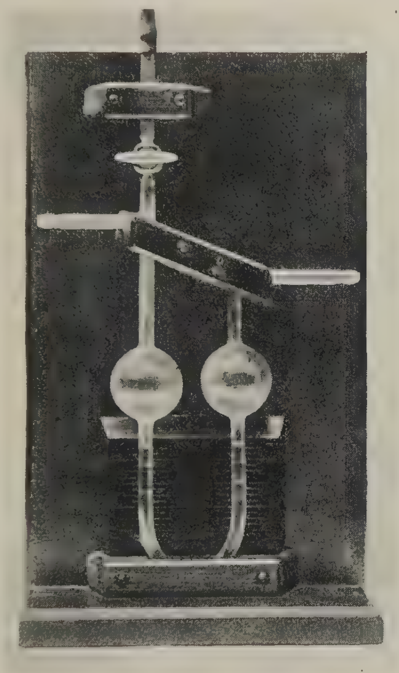



FIG. 62.—Leak tester.

As regards choice of rubber tubing, it will be found that for general use fairly thick-walled tube of red rubber, having a bore of $\frac{3}{16}$ inch and an external diameter of about $\frac{3}{8}$ inch will be the most suitable. Such tubing will usually be found to be non-porous and considerably more permanent than thin black tubing, which is not to be recommended for this type of work. It is probably also to be preferred to the thick-walled pressure tubing on account of the greater ease of making connections.

**Theory of the Curved Tube Manometer.**—We shall assume that the manometer is intended to be used with a standard Pitot and static tube. If, then, $p$ is the pressure difference in pounds per square foot that this instrument transmits to the manometer when used in a wind of $v$ feet per second, we have the equation

$$p = \tfrac{1}{2}\rho v^2.$$

If $h$ is the corresponding height in inches through which the level of the manometric liquid rises above its zero level (we shall take the reservoir to be sufficiently large to warrant the assumption that all the change in level occurs in the curved tube),

$$5{\cdot}2\,\frac{h}{\sigma} = \tfrac{1}{2}\rho v^2,$$

where $\sigma$ is the specific gravity of the manometric liquid.

Hence $$v\sqrt{\rho} = \sqrt{\frac{10{\cdot}4h}{\sigma}}. \quad . \quad . \quad . \quad (8)$$

In Fig. 63 let O be the zero position of the liquid meniscus against the upper side of the curved tube, and take O as the origin of co-ordinates. The condition to be fulfilled is that

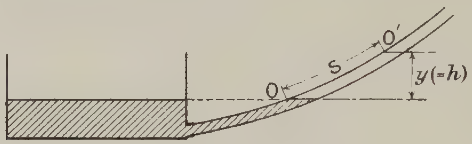



FIG. 63.

for a given increment in $v\sqrt{\rho}$ the meniscus is to move a given distance along the tube to O′. Hence the equation of the curve of the tube is

$$s = \mathrm{A}v\sqrt{\rho}, \quad . \quad . \quad . \quad . \quad (9)$$

where $s$ is the distance moved measured along the tube from O and A is a convenient constant.

Hence, from (8),

$$s = \mathrm{A}\sqrt{\frac{10{\cdot}4}{\sigma}\,h} = \mathrm{B}\sqrt{h}.$$

If $x$ and $y$ are the co-ordinates of O′, measured from the origin O,

$$s = \mathrm{B}\sqrt{h} = \mathrm{B}\sqrt{y}.$$

Differentiating, we have $ds = \dfrac{\mathrm{B}}{2\sqrt{y}}\,dy,$

i.e. $$\sqrt{dx^2 + dy^2} = \frac{\mathrm{B}}{2\sqrt{y}}\,dy,$$

or $$\frac{dx^2}{dy^2} = \frac{\mathrm{B}^2}{4y} - 1.$$

Hence $$dx = \sqrt{\frac{\mathrm{B}^2}{4y} - 1}\;.\;dy,$$

and the curve to which the tube is to be bent is obtained by integrating this equation. This gives

$$x = \int \sqrt{\frac{B^2}{4y} - 1}\, dy + C.$$

The right-hand side is easily evaluated by the substitution $y = \frac{B^2}{4} \sin^2 \theta$, and the final expression will be found to be

$$x = \frac{B^2}{4} \sin^{-1} \sqrt{\frac{4y}{B^2}} + \frac{B}{2} \sqrt{y\left(1 - \frac{4y}{B^2}\right)}. \qquad (10)$$

The constant of integration C is found to be zero by putting $x$ and $y$ equal to 0 simultaneously at the origin.

If, therefore, the axis of the tube is bent to the form represented by equation (10), the gauge will show equal movements of the liquid meniscus along the tube for equal changes in $v\sqrt{\rho}$. Instruments constructed on these lines require calibration against a standard manometer, just as in the case of the ordinary inclined tube gauge.

It is clear from the above analysis that if the instrument is furnished with a velocity scale, this can only be accurate at one value of the air density $\rho$, which must be specified. For any other density $\rho'$, the readings must be multiplied by $\sqrt{\frac{\rho}{\rho'}}$; that is, if the scale is said to be accurate for air (assumed dry) at a temperature T absolute and at a pressure $p$, then the readings for a temperature T′ and a pressure $p'$ must be multiplied by $\sqrt{\frac{T'p}{Tp'}}$.

# CHAPTER X.

## METHODS OF FLOW MEASUREMENT BASED UPON THE RATES OF COOLING OF HOT BODIES.

THE relation between the rate of loss of heat of a heated body and the speed of flow of a fluid in which it is immersed has formed the subject of a considerable amount of research. Attention has mainly been concentrated on problems connected with the use of electrically heated wires as anemometers, but recently a new line of investigation has been developed in which a specially constructed alcohol thermometer is employed in a similar manner. The laws governing the convective cooling of hot cylindrical wires by streams of fluids are now, empirically at all events, fairly well understood, and measurements of air flow have successfully been made in large numbers of cases by methods based on these laws. Whilst such methods possess definite advantages, chief among which is the possibility of obtaining high sensitivity for low rates of flow, certain manipulative difficulties have hitherto prevented them from coming within the domain of general practical anemometry. Nevertheless, for special work, particularly under laboratory conditions, instruments utilising these principles may be made to yield excellent results, and for this reason it has been thought desirable to include a comparatively brief discussion of the subject here. Further information may be obtained by reference to the original papers quoted in the bibliography at the end of this chapter, and to other sources cited in these papers themselves.

**The Cooling of Heated Wires.**—The most extensive researches into the laws of cooling of electrically heated wires are probably those of L. V. King.* Developing the theoretical treatment initiated by Boussinesq,* King showed that the relation between the heat lost H per unit length of a cylinder of diameter $d$, maintained at a temperature T above that of the

* See bibliography, p. 195.

fluid in which it is immersed, could be represented very nearly by the equation

$$H = KT + \sqrt{2\pi KS\sigma d}\ V^{\frac{1}{2}}T, \quad . \quad . \quad (1)$$

where K is the thermal conductivity of the fluid, S its specific heat at constant volume, $\sigma$ its density, and V its velocity.

King showed that this expression might be expected to hold down to a value of $Vd$ of 0·0187, V being expressed in centimetres per second and $d$ in centimetres, which is equivalent to a limiting speed of about $\frac{1}{4}$ foot per second for a wire of diameter 0·001 inch. It should be noted—and attention is drawn to this point by King—that in deriving the theoretical relationship (1) it is assumed that V is the total speed of the fluid past the wire. Now a heated wire placed in air will give rise to convection currents which will assist in the cooling of the wire, and at very low velocities these currents may have an appreciable effect in modifying the true speed of the air past the wire. It is probable therefore that in practice the lower limit of the speed range over which (1) is applicable will be somewhat higher than its value as given by the above-mentioned criterion.

Equation (1) may be written in the form

$$H = B\sqrt{V} + C, \quad . \quad . \quad . \quad (2)$$

where B and C are functions of temperature and the properties of the fluid and the wire mentioned above. For a wire of given dimensions, maintained at a constant excess temperature, and always used in the same fluid (air, for example) B and C are constants, and may be calculated for the prescribed conditions. In order to verify his theoretical deductions, King undertook a comprehensive experimental investigation of the heat losses of electrically heated platinum wires ranging in diameter from about 0·001 inch to 0·006 inch when moved, on a rotating arm, through still air at speeds between about 0·5 and 29 feet per second. He found that, within these limits, the results could be accurately expressed by an equation of the form of (2), and further work carried out by him and other investigators has shown that this equation holds also for speeds up to at least 100 feet per second. The actual values of B and C determined by King were in moderately good agreement with those calculated from theory.

**The Use of Electrically Heated Wires in Anemometry.**—If a current $i$ ampères is flowing in a wire of resistance

R ohms the heat generated per second is equal to $\frac{i^2R}{J}$ calories, where J is the mechanical equivalent of heat in joules per calorie. Hence for an electrically heated wire equation (2) becomes

$$\frac{i^2R}{J} = B\sqrt{V} + C. \qquad . \qquad . \qquad . \qquad (3)$$

Now if the temperature of the wire is maintained constant, its resistance R will also be constant, and if we confine our attention to a particular fluid, (3) becomes

$$i^2 = K_1\sqrt{V} + K_2, \qquad . \qquad . \qquad . \qquad (4)$$

where $K_1$ and $K_2$ are constants for a given wire.

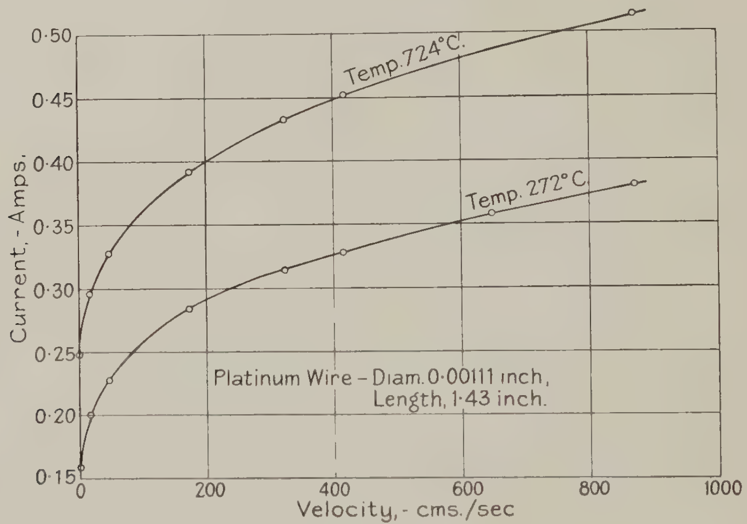



Fig. 64.

This relation between current and air speed for a wire maintained at a constant temperature in air is shown graphically in Fig. 64, which is taken from King's results. The upper curve was obtained with the wire at a temperature of 724° C., whilst for the lower curve the temperature was 272° C. These curves show well the special characteristic of the hot-wire anemometer, namely, the high degree of sensitivity at low wind speeds. It will be seen that the rate of change of current with wind speed is much more rapid at the lower speeds than at the

higher, and also that the higher the temperature of the wire, the higher is the sensitivity over the whole range.

Turning once more to equation (4), we see that if $i_0$ is the value of the current necessary to maintain the wire at a prescribed constant temperature in still air (i.e. when $V = 0$), $K_2 = i_0^2$ and we may write

$$i^2 = i_0^2 + K\sqrt{V} \quad . \quad . \quad . \quad (5)$$

as the relation between current and wind speed in its most useful practical form. The heat loss represented by $i_0^2R$ is the total loss due to radiation and convection in still air. For the reasons stated above, when V is so low that it becomes comparable with the speed of the free convection currents due to the hot wire in still air, equation (5) requires modification, but for ordinary purposes this will not be necessary. Some calculations made by King indicate that for a wire 0·003 inch diameter the free convection current has a velocity of the order of $\frac{1}{2}$ foot per second when the wire is at 1000° C., and about $\frac{1}{4}$ foot per second at 200° C. If, therefore, a wire is to be used to measure low speeds of magnitudes comparable with these, the calibration should be determined from tests at such speeds.

**Effect of Variation of Atmospheric Conditions.**—The variable most affecting the relation between the current in a given wire and the wind speed is atmospheric temperature. From what has been stated above, it will be clear that equation (5) only holds when the wire is maintained at a constant temperature, and that the values of $i_0$ and K, once determined, only relate to a definite *excess* temperature of the wire over the surrounding air. Any change in the air temperature will therefore produce a change in the calibration curve of a given wire. Changes in air density due to variations of atmospheric temperature also need consideration. If we assume that the other quantities occurring in equation (1), viz. the specific heat and conductivity of the air, are not affected by variations in atmospheric temperature, and if we retain T as a variable, equation (1) may be reduced to the form

$$i^2R = C_1(T_w - T_a) + C_2(T_w - T_a)\sqrt{V\sigma}, \quad . \quad (6)$$

where $T = T_w - T_a$ = wire temperature − atmospheric temperature,

$$C_1 = \text{constant} = \frac{i_0^2R}{T_w - T_a},$$

and $C_2$ = constant.

Equation (6) may be written

$$\sqrt{V\sigma} = \frac{i^2R - C_1(T_w - T_a)}{C_2(T_w - T_a)} \quad . \quad . \quad . \quad (7)$$

Treating $T_w$ as constant (hence R is also constant) and $(V\sigma)$ as a single variable, and differentiating with respect to $T_a$, we have

$$\frac{1}{2\sqrt{V\sigma}} \frac{d(V\sigma)}{dT_a} = \frac{i^2R}{C_2(T_w - T_a)^2}. \quad . \quad . \quad (8)$$

Equations (7) and (8) may be combined to give, when $\dfrac{i_0^2R}{T_w - T_a}$ is written for $C_1$,

$$\frac{d(V\sigma)}{(V\sigma)} = \frac{2dT_a}{T_w - T_a} \frac{i^2}{(i^2 - i_0^2)}, \quad . \quad . \quad (9)$$

which shows the fractional change in $V\sigma$ for a change in air temperature given by $dT_a$.

For a platinum wire 0·0025 inch in diameter employed at 1000° C. King gives $i_0^2 = 0{\cdot}5$ (amps.)$^2$ approximately, whilst for a range of velocities from 16·4 to 92 feet per second $i$ varied between 1 and 2 ampères. If these values are substituted in (9) it will be seen that a change of air temperature of $\pm$ 2°C. produces a change in $(V\sigma)$ of less than 1 per cent. over this range of air speed. If the air temperature changes by more than this amount a correction can easily be applied by the use of (9).

The other effect of a variation in $T_a$, to which reference has already been made, is the accompanying change in the density $\sigma$. In this it is associated also with changes of barometric pressure and humidity. The equations already given show that the hot wire really measures the product of velocity and density. If therefore a given wire is calibrated in terms of velocity, this calibration will only hold for a given value of $\sigma$. It will readily be seen from equation (6) that if V is the velocity corresponding, according to the calibration curve for density $\sigma_0$, to a current $i$, the same current will correspond, when the density is $\sigma_1$, to an air speed $V\dfrac{\sigma_0}{\sigma_1}$. The densities $\sigma_0$ and $\sigma_1$ are easily calculable from observations of air temperature and barometric pressure. Relative humidity may also be taken into account if desired, but its effect will rarely be important.

In the above analysis it has been assumed that changes of atmospheric conditions do not affect the other properties of the air appearing in equation (1), viz. the specific heat and the thermal conductivity. This assumption will be true for all practical purposes even under extreme variations in natural conditions, but for special work, such as, for example, the measurement of the flow of hot air, the possibility of variations in these properties should not be overlooked.

**Methods of Measurement in Hot-Wire Anemometry.**—It has been shown above that if a wire is maintained at a constant temperature (and consequently also at a constant resistance) the current varies with the wind speed according to the law expressed by equation (5). Alternatively, if the current passing through a wire is kept constant its temperature and resistance will vary with the wind speed in a definite manner. There are, accordingly, two possible methods of measurement, which we may term respectively the constant resistance method, and the constant current method. Experiments by Simmons and Bailey * show that, except at low wire temperatures (less than about 150° C.), the constant current method leads to higher sensitivity. In practice the constant resistance method is, however, probably to be preferred, partly on account of the fact that it is somewhat easier accurately to maintain a constant resistance than a constant current, and partly because the constant K in equation (5) can then be determined by a calibration at a single air speed, $i_0$ being the current necessary to bring the wire to the desired resistance in still air. On the other hand, the constant current method allows, as we shall see, a direct reading of wind speed to be obtained.

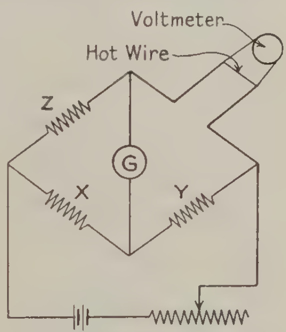


FIG. 65.

(*a*) *Constant Resistance Methods.*—In the simplest case, the hot wire forms one arm of a Wheatstone bridge—see Fig. 65, the opposite arm Z consisting of a manganin resistance having practically zero temperature coefficient. The hot wire is of some pure metal, such as nickel or platinum, whose electrical constants are accurately known; the resistances forming the

* See bibliography, p. 195.

other three arms of the bridge can accordingly be arranged so that, when the bridge is in balance, the resistance, and consequently the temperature, of the hot wire is at its desired value. Any change in the speed of the air current past the hot wire will then necessitate a change in the heating current in order to restore the temperature of the wire to its former value. When the wire is again at this temperature the bridge will once more be in balance, and the change in current will be a measure of the wind speed.

The current may be measured in a number of ways. An ammeter of the requisite sensitivity and range may be connected in series with the hot wire, so that the current passing through it can be measured directly, or a high resistance voltmeter connected across the wire may be used as a measure of the current, and hence of the wind speed, since the wire always has the same resistance. Alternatively, the current in the external circuit may be measured by an ammeter. For the highest accuracy the scheme illustrated in Fig. 66, which was used by Simmons and Bailey,* is recommended. Here the current in the external circuit is measured by a potentiometer method; the voltage drop across a standard known resistance R, having a negligibly small temperature coefficient, is determined, and the current flowing can easily be calculated. The standard cell shown in the diagram is used to calibrate the potentiometer directly in terms of voltage. Since no current flows when balance of the galvanometer $G_2$ is obtained during standardisation of the potentiometer, the voltage across the portion AB is equal to the known voltage of the standard cell. A small variable rheostat, as shown, is connected in series with the potentiometer cell in order that balance of the standard cell may always be obtained at the

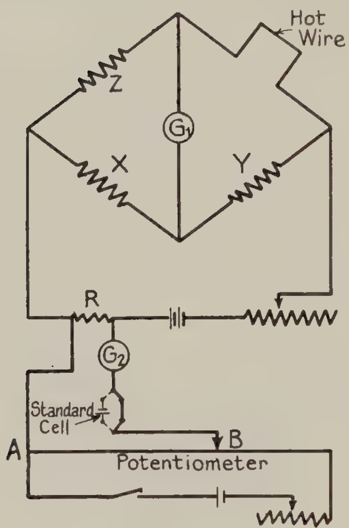



Fig. 66.

* See bibliography, p. 195.

same potentiometer reading, in spite of fluctuations in the voltage of the potentiometer cell.

The usual arrangement of the arms of the bridge is to make the two ratio coils X and Y of equal resistance, and the manganin resistance Z equal to that of the hot wire when the latter is at its desired temperature. This value can of course be calculated beforehand from a knowledge of the temperature-resistance coefficient of the material of which the wire is composed. The ratio coils should be of the same material, preferably manganin.

As regards the temperature of the hot wire, it follows from equation (9) that the higher the temperature excess $T_w - T_a$, the smaller will be the errors due to changes of atmospheric temperature. If, therefore, it is desired to avoid the necessity for applying temperature corrections, it will be necessary to run the wire at a high temperature. King advocated a working temperature of 1000° C., for which, as we have seen, the errors in the measurements of V will not often exceed 1 per cent., if the initial calibration is performed at, say, 17° C. In order to maintain a high working temperature such as this, fairly large currents are required at the higher wind speeds, and in these circumstances it may be desirable to use, as King did, a Kelvin double bridge in place of the ordinary Wheatstone net. A description of this form of bridge, which allows large currents to be passed through the anemometer wire without overloading the ratio coils, will be found in any text-book dealing with electrical measurements. It has the additional advantage of high sensitivity, and that its readings are, to the order of accuracy required for hot-wire anemometry, independent of contact and lead resistances. The latter feature is of importance for some types of work, as it allows the hot wire to be mounted on a moving body, the current being led to it through brush contacts or slip rings.

Although the use of a high working temperature has the merits of increased sensitivity and the reduction of errors due to small changes of air temperature, a temperature as high as 1000° C. is not to be recommended for general purposes. In the first place, there is a risk of fusing the anemometer wire by the accidental passage through it of a comparatively small excess of current, especially when working at low wind speeds. Secondly, there is an "ageing" effect, mentioned by King and found by Simmons and Bailey * to be quite appreciable

* See bibliography, p. 195.

with fine wires (0·001 inch diameter). This effect causes progressive changes in the calibration of a given wire, and there is evidence to show that it is more important the higher the working temperature. It seems, therefore, preferable to work with the anemometer wire at some lower temperature, say 400° C. to 500° C., and either to correct if necessary for atmospheric temperature variations by the use of equation (9), or to adopt some form of circuit which automatically compensates for such variations.

King described a method * by which the readings obtained from the Kelvin double bridge can be made to indicate velocity independently of the air temperature, to the degree of accuracy required in practice. Certain of the ratio coils, instead of being made of manganin, are made of the same material, in this case platinum, as the hot wire, so that they have the same temperature coefficient of resistance. These coils are exposed to the airstream so that they can readily attain its temperature.

A method of temperature compensation suggested by Davis * is illustrated in Fig. 67. The three resistances X, Y, and Z are of manganin, whilst in the arm of the bridge containing the resistance Z there is a second coil P which can be short-circuited by means of the key K. The connections are so arranged that when this coil is short-circuited the current through the bridge is automatically decreased about one hundredfold, so that only a very small current passes through the anemometer wire. The latter now acts as a platinum resistance thermometer, and by moving the contact C along the slide wire SS the bridge is balanced at a definite position depending on the temperature of the air flowing past the anemometer wire. If required, a scale of air temperature may be marked along SS. The contact is fixed at the balance-point on SS, and the key is now raised. This process increases the resistance of that arm of the bridge by the resistance P, which is constructed of manganin and is therefore unaffected by ordinary temperature changes, and simultaneously sends a much larger current through the whole bridge, so that the hot wire is heated. This current is then adjusted by the rheostat until balance is again obtained. It will be clear that the resistance of the anemometer wire is then increased by an amount equal to the resistance of P. If the resistance-temperature law for the anemometer wire is linear it follows that the wire is always heated to a constant

* See bibliography, p. 195.

temperature excess above the air temperature, and, actually, the departure from this condition is not sufficient to cause appreciable errors. For his anemometer wire Davis used platinum, 0·006 inch diameter, heated to 108° C. above atmospheric temperature.

A valuable method of measurement, due to Professor Callendar, has been described by R. O. King.* In series with the external circuit of the Wheatstone bridge, is a device called a volt-thermometer (Fig. 68), which consists simply of a length of fine platinum wire whose function is described below. In the bridge itself, the two ratio coils X and Y are of manganin, having resistances of 100 ohms and 400 ohms respectively, whilst the standard resistance Z is 1 ohm

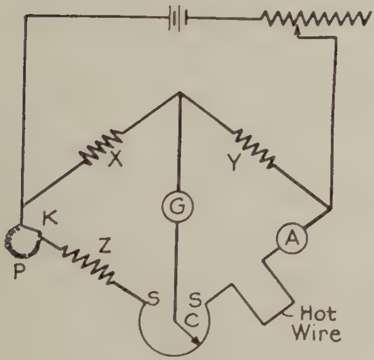



FIG. 67.

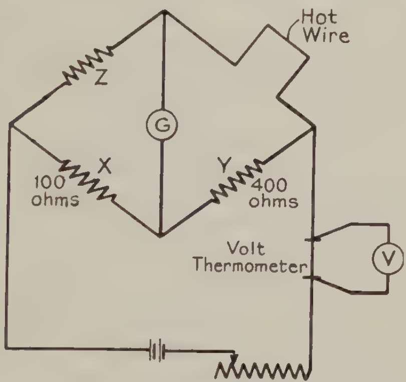



FIG. 68.

and the anemometer wire 4 ohms when hot. It will be clear therefore that no appreciable error is made in assuming that the current through the volt-thermometer is equal to that through the anemometer wire. Instead of observations of this current, measurements of the voltage drop across the volt-thermometer are made. As the external current is altered to maintain balance of the bridge, so the heating effect of this current through the volt-thermometer wire changes its temperature, and consequently its resistance and the voltage drop across it. Callendar has shown experimentally that over a considerable range there is a linear relation between the resistance R of the wire and the voltage drop V, and that for a

* See bibliography, p. 195.

3-inch length of 0·004 inch diameter platinum wire this relation can be written

$$R = 0{\cdot}607V + 1 \text{ ohm.} \qquad (10)$$

Hence, if C is the current through the anemometer wire, since this current may be taken as flowing also through the volt-thermometer, we have

$$C = \frac{V}{R} = \frac{V}{0{\cdot}607V + 1}. \qquad (11)$$

Plotting this relation between C and V (Fig. 69), we see that V increases rapidly with C, the rate of increase being, in fact, somewhat faster than the square of the current. The physical significance of this is that as the current in the wire is increased its temperature rises rapidly. Its resistance therefore increases at a proportionate rate, so that a rapidly rising voltage drop occurs across the wire. This point is well illustrated by the curve of Fig. 70, given by Simmons and Bailey,* which shows the rapid rise of temperature with heating current in still air of a platinum wire 0·00105 inch in diameter and 3·07 inches in length.

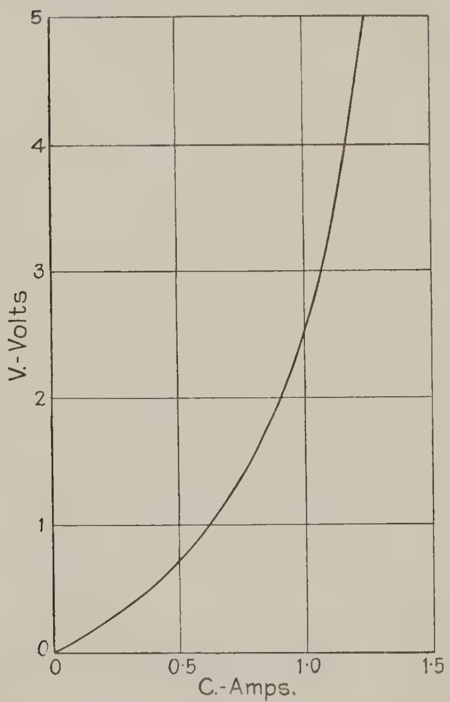



FIG. 69.

It will be seen therefore that, whereas in the ordinary method of measurement the air speed varies, according to equation (5), approximately as the fourth power of the quantity observed, viz. the current in the anemometer wire, it can be made to vary as some power between 1 and 2 of the observed reading if the latter is the voltage drop across the volt-ther-

* See bibliography, p. 195.

mometer wire, instead of the heating current. A valuable increase in accuracy is thus afforded by this method. A further important feature is the possibility of extending considerably the range of air speed that can be observed with one measuring instrument—in this case, a voltmeter. For if two similar volt-thermometer wires are used in parallel in place of the single wire, only half the total current will flow through each, and consequently a full scale deflection of the voltmeter connected across these wires will correspond to twice the total current passing when one wire is used. That is, since the wind speed is approximately proportional to the fourth power of the current, a given voltmeter deflection will correspond to roughly $2^4$ or 16 times the wind speed which it previously indicated. It is claimed that the accuracy attainable is well within 1 per cent. for all speeds up to about 300 feet per second. The factor limiting the indefinite extension of the scale in this way will be the maximum current that can safely be passed through the circuit.

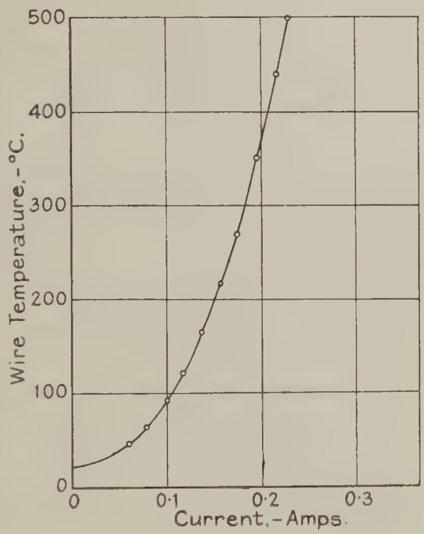


Fig. 70.

R. O. King * has described an actual anemometer working on these lines. The anemometer wire was of 0·004 inch diameter platinum having a total length of 6 inches, arranged in four sections. The volt-thermometer consisted of four platinum wires, each 0·004 inch in diameter and 3 inches long, with the necessary connections so that one wire alone, or two, three, or four in parallel, could be used as required to give the maximum accuracy for any particular air speed being measured. It was found that a single platinum wire of these dimensions would safely take a current producing a 5-volt drop across it. In order to prevent the loss of heat by convection from the

* See bibliography, p. 195.

volt-thermometer and to protect it from draughts, its four wires were enclosed in a fairly air-tight box. This arrangement was found to be satisfactory up to values of the total current of about 2 or 2½ ampères. For measurements of high rates of flow, where the current attained the order of 5 ampères, it was found necessary to water-jacket the box containing the volt-thermometer wires in order to maintain sufficiently constant temperature conditions at the wires. A five-range voltmeter, reading up to 5 volts, was used with this anemometer.

(*b*) *Constant Current Method.*—A hot-wire instrument based upon this method of measurement has been designed by MacGregor Morris, and has been put on the market for the measurement of low air speeds. Fig. 71 shows the essential features of the circuit used. The anemometer wire A is of nickel, 0·005 inch in diameter and about 6 inches long, having a cold resistance of about 0·9 ohm. B is a similar nickel wire, and C and D, which form the other arms of the bridge proper, are manganin resistances of 1 ohm each. The galvanometer used in the constant resistance circuits as a null reading instrument to indicate balance of the bridge is now replaced by the reading milli-ammeter M. If we assume for the moment that the wires A and B are exactly similar, as are also the resistances C and D, we see that the current in the external circuit will always divide into two currents equal in magnitude through the arms BC and AD. The bridge will be in balance in still air when the resistances of A and B are made equal to those of D and C, that is, when the current in each branch heats the nickel wires sufficiently to raise their resistances from 0·9 to 1·0 ohm. No current will then flow through the milli-ammeter. If now the wire A is exposed to the wind, its resistance will drop and the bridge will no longer be balanced. The milli-ammeter then registers an "out-of-balance" current, which is utilised as a measure of the wind speed.

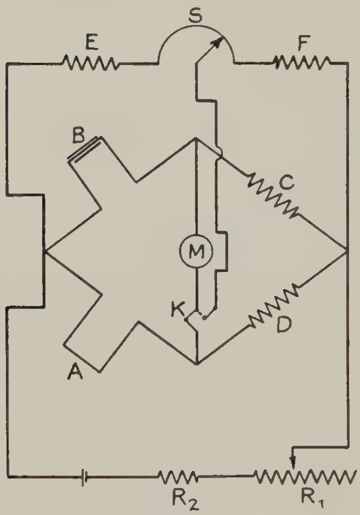


Fig. 71.

A special feature of this instrument is the method of compensation against variations of atmospheric temperature. All four wires forming the arms of the bridge ABCD are carried in a frame which is exposed to the air current to be measured. The anemometer wire A is, of course, bare, but the other nickel wire B is shielded by a thin metal tube from the convective cooling of the stream. By means of a simple change-over switch the shielded wire B and the manganin resistance C can be made part of an auxiliary bridge circuit whose other two arms are the manganin resistances E and F of about 20 ohms each. The junction on the bridge between E and F can be varied over the range of the slide wire S, which has a total resistance of about 5 ohms, so that the ratio of the total resistances of the arms E and F can be accordingly varied. The process of switching over to the auxiliary bridge connections automatically connects the milli-ammeter between the junctions of E and F and of B and C. At the same time a large additional resistance is put into the external circuit so that the current through the wire B is reduced to a sufficiently small value to ensure that it produces no appreciable rise of temperature of this wire over that of its surroundings. In this condition B acts as a resistance thermometer, and the auxiliary bridge is balanced at zero on the milli-ammeter scale by moving the adjustable contact over the slide wire S. The balance-point will depend on the temperature of the wire B, that is, in effect, since the tube enclosing this wire is exposed to the air current, on the prevailing air temperature.

A scale of temperature is marked along the slide wire, and the temperature reading at which balance is obtained is noted. The bridge is then thrown out of balance by moving the contact on the slide wire through a definite temperature range, which varies slightly with each instrument, of about 40° F. This alters the ratio of E to F so that balance of the auxiliary bridge can only be restored by sending an increased current through the nickel wire B, sufficient to raise its resistance by an amount equivalent to a definite temperature excess (about 40° F.) over that of the air. With the contact on S in its new position, therefore, the high series resistance in the external circuit is cut out and the current is adjusted by the rheostat R, until balance is again obtained. The instrument is now adjusted ready for use, and the change-over switch is put into the position which brings the main bridge ABCD into the circuit. If the wires are accurately adjusted so that the

pairs A and B, C and D are exactly similar, the bridge will be in balance when the air speed past A is zero,* but when the air is moving A will be cooled, whereas B, being shielded, will not ; further, C and D, being of manganin, will not alter in resistance. An out-of-balance current will therefore flow through the milli-ammeter, which can be calibrated in terms of wind speed.

If the wires A and B are not exactly similar there will be a slight zero error on the instrument, i.e. a small milli-ammeter deflection at zero wind speed. This, however, can easily be eliminated by arranging the temperature excess to which B is initially set, to be such that when the main bridge is in the circuit the resistances of A and B will be equal, so that the current through A will be equal to that through B. Thus, by a simple alteration of the balance-point on S, which will constitute a constant for the instrument, the main bridge can always be made to balance in still air. The wires A and B will then be at slightly different temperatures, but the difference between their temperatures in still air will be constant. It is for this reason, namely, that the wires A and B are not always exactly similar, that the temperature excess to which the auxiliary bridge is to be balanced varies somewhat from one instrument to another. Although there is less difficulty in making the manganin resistances C and D equal, any slight dissimilarity in these can be allowed for by the same means.

An instrument having resistances and dimensions of wires of the magnitudes mentioned above is suitable for measuring velocities up to 200 feet per minute. A photograph of such an anemometer is reproduced, by the courtesy of Messrs. H. Tinsley & Co., Ltd., the manufacturers, in Fig. 72. The frame carrying the bare and shielded nickel wires and the two manganin resistances forming the other arms of the bridge will be seen to the left of the diagram, the milli-ammeter and the controlling switch and resistances being conveniently arranged in the box on the right. The anemometer wire is always held in a vertical position when in use, so that natural convection currents always act in the same direction relatively to the wire. Calibration of the instrument is also performed with the wire vertical. A two-volt accumulator, fitting into a special compartment in the box, is supplied with the instrument. A small fixed resistance ($R_2$ in Fig. 71), permanently in series

* Apart from very small errors due to natural convection currents from A.

with the battery circuit, is provided, to prevent accidental short-circuiting of the cell.

**Notes on Anemometer Wires.**—Platinum has been more extensively used for anemometer wires than any other metal, chiefly because of its chemical inertness to atmospheric conditions even at high temperatures. For low temperature wires, such as those used by MacGregor Morris, nickel has been found to be suitable, provided it is used in a high state of purity. Electrolytically prepared nickel gives satisfactory results.

The dimensions of the wire are of some importance. It

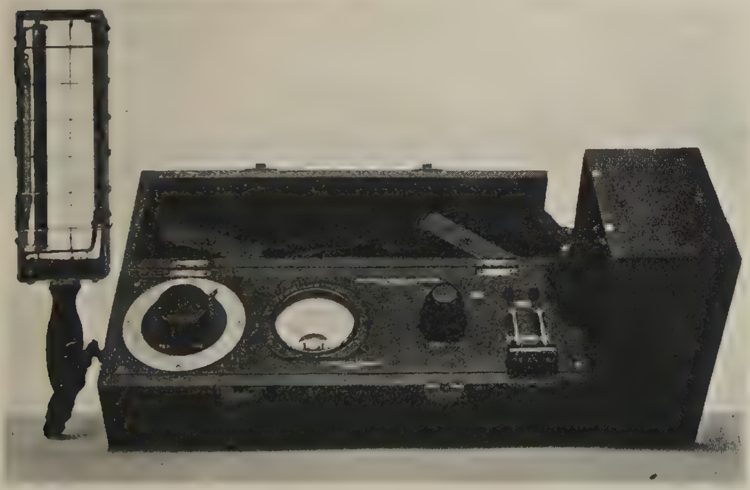


FIG. 72.—The MacGregor Morris hot-wire anemometer.

would seem that fine wires are more prone to the ageing effect, to which reference has been made previously, than stouter wires. From this point of view, therefore, it is desirable to have wires of large diameter. There is, however, the objection to thick wires that large currents are needed to raise them to the desired temperature ; few, if any, experimenters have used wires exceeding 0·006 inch in diameter.

With regard to length, too short a wire will not give the requisite sensitivity, but a long wire may become somewhat slack when heated and its natural period of vibration may become comparable with the frequency of the eddies generated by the motion of the air past it. There will then be a tendency

for the wire to take up an oscillatory motion, which will increase the rate of cooling and cause a spurious velocity reading. If the amplitude of vibration is at all large, the errors due to this cause may be appreciable. In order to avoid risk of such effects it is best to limit the length of the wire and to mount it in such a manner that it is always under tension, which may be conveniently provided by arranging for a certain amount of spring in the wire supports, tending to keep the wire taut. The length of wire is also determined to some extent by the use to which the instrument is to be put. The method of support should be designed to introduce as little disturbance of the airstream as possible, especially if the instrument is to be used in relatively small pipes. Ordinary solder should never be used for attaching the wire to its supports. Hard silver solder is suitable except for very high wire temperatures, when it is better to fuse or weld the wire to its end connections. Before use, a new wire should always be annealed by passing a current through it so as to raise its temperature to red heat for some time. For platinum wires King advocated a period of annealing of some hours' duration. Finally, the calibration should be checked at frequent intervals, particularly if the wire is used at a high temperature.

**Relation between the Temperature and Resistance of a Wire.**—It is necessary, when designing a bridge to suit a given wire, to make calculations of the approximate resistance of the wire. The resistance of a circular wire of length $l$ inches and diameter $d$ inches at 0° C. is equal to

$$\frac{4lk}{\pi d^2} \text{ ohms},$$

where $k$, the specific resistance, may be taken as $4{\cdot}2 \times 10^{-6}$ for platinum and $4{\cdot}6 \times 10^{-6}$ for nickel.

The resistance $R_t$ at any other temperature T° C. is given with sufficient accuracy by the equation

$$R_t = R_0(1 + \alpha T), \quad . \quad . \quad . \quad (12)$$

where $R_0$ = resistance at 0° C., and $\alpha = 38 \times 10^{-4}$ for platinum and $62 \times 10^{-4}$ for nickel.

In calculating the temperature of a wire from its measured resistance by the use of this equation, small errors are introduced by the assumption that $\alpha$ is strictly constant, and by neglect of the heat loss by conduction to the wire supports.

For approximate calculations, however, this relationship will give sufficient accuracy.

**Electric Flow Meters.**—Thomas * and Callendar * have designed electrical meters which indicate the rate of flow of air from measurements of the heat energy which must be supplied to the moving stream, at a specified position, in order to maintain a definite difference of temperature between two given sections on either side of that at which the heat is supplied. If $t°$ C. is the constant temperature difference between these sections, the weight of air W flowing per second is given by

$$W = \frac{H}{st} \text{ grams per second,} \qquad . \qquad . \quad (13)$$

where H is the heat supplied in calories per second and $s$ is the specific heat of air.

If this heat is supplied by an electric current C ampères flowing through a resistance R ohms placed in the air stream, equation (13) becomes

$$W = \frac{C^2R}{Jst}, \quad . \quad . \quad . \quad . \quad (14)$$

where J is, as before, the mechanical equivalent of heat in Joules per calorie. Alternatively, if V is the voltage drop across the resistance, we may write

$$W = \frac{V^2}{RJst}. \quad . \quad . \quad . \quad . \quad (15)$$

In Callendar's instrument use is made of the relationship in the form of (15). In brief, the apparatus consists of a resistance coil JK (Fig. 73) of material having a constant resistance, made in the form of a mat extending over the cross-section of the airstream to be measured, so that the latter is uniformly heated.

The temperature rise is measured by means of two resistance thermometers BD and EC, consisting of grids of fine platinum wire placed at equal distances on either side of the heating mat. These thermometers form part of the Wheatstone bridge circuit BACEFD, in which F is the balance point when no current is passing through the heating grid, i.e. when both thermometers are at the same temperature. When the air current is flowing in the direction shown, and the grid is heated,

* See bibliography, p. 196.

EC is hotter than BD and its resistance is higher. A new balance-point may be obtained at H on a slide-wire resistance between D and E. The resistance FH is made equal to the difference in resistance of the two thermometers when their temperatures differ by a fixed amount—$2\frac{1}{2}°$ C. was used in an actual case—and the heating current through the grid JK is adjusted to maintain this temperature difference constant, i.e. to keep the bridge BACEFD balanced. In equation (15) we then have the numerator constant, so that W varies as $V^2$.

The voltage drop across the heating grid, which is thus a measure of the wind speed, is determined by an arrangement similar to the volt-thermometer previously described. The

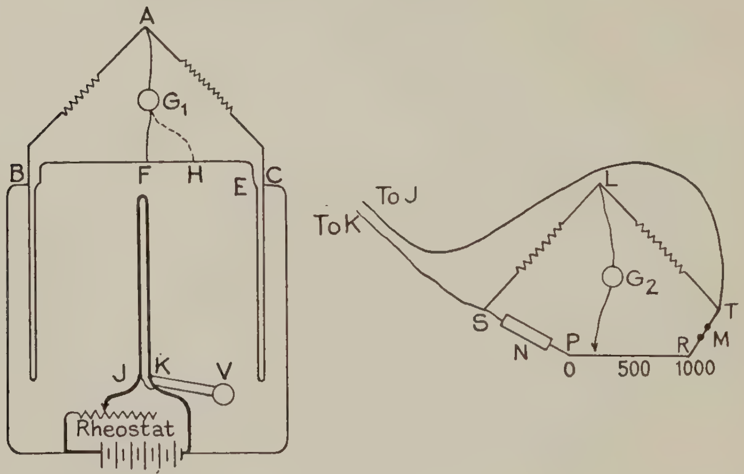



FIG. 73.

leads from the terminals JK of the grid are connected to a second Wheatstone bridge SLTRP, of which the arms SL and LT are equal resistances, having zero temperature coefficient. The arm TR contains the volt-thermometer M, which is, as before, simply a fine platinum wire enclosed in a box, whilst N is a balancing resistance. PR is a potentiometer wire, to any point of which the connection of the galvanometer $G_2$ may be made. Instead of calibrating the bridge in terms of the distance of the balance-point from P when known voltages are applied, the rate of air flow may be correlated directly by calibration with the position of the balance-point. The method of operation in taking an observation is to adjust the rheostat

in the heating circuit of JK until the galvanometer $G_1$ indicates balance of the thermometer bridge with the connection at H, and then to find the balance-point for the galvanometer $G_2$ on the potentiometer PR.

Complete descriptions of the instrument and of measurements conducted with it are given in the paper quoted in the bibliography, and reference may be made to this source for further details.

In the Thomas meter * a similar arrangement of heating and thermometer coils is used, but in this case the power supplied to the heating circuit is measured directly on a wattmeter. In an automatic form of this instrument, which has been put on the market for commercial use, an ingenious device is introduced to avoid the necessity for adjusting the current through the heating coil by hand, in order to maintain the thermometers at a constant temperature difference. By means of an electric motor drive, the needle of the galvanometer in the thermometer bridge is clamped between contacts at regular intervals of a few seconds. If the temperature difference is not at its prescribed value when one of these periodic contacts is made, the galvanometer needle will be off its zero position and a relay circuit is made which automatically varies a rheostat in the heater circuit, and so increases or decreases the current in this circuit, according to whether the temperature difference is too low or too high. The amount by which the resistance is varied is made to depend upon the deflection of the galvanometer in the bridge circuit, that is, upon the amount by which the temperature difference between the two thermometers has deviated from its standard value.

**The Kata-Thermometer.**—In the course of his researches on atmospheric conditions as affecting health and industrial efficiency, Dr. Leonard Hill designed an instrument known as the Kata-Thermometer to measure the rate of heat loss of a surface at approximately body temperature with variable atmospheric conditions. This instrument, which depends in principle upon the rate of cooling of a hot body, is essentially a specially designed alcohol thermometer. The bulb consists of a cylinder, 2·2 cms. in length and 1·8 cms. in diameter, with hemispherical ends, the total length being therefore 4 cms. Leading from the top of the bulb is a stem upon which is marked the range of temperature 100° F. to 95° F. The bore of the

* See bibliography, p. 196.

stem is enlarged at its upper end, so that the instrument may be heated considerably above 100° F. without risk of fracture.

If an instrument of this kind is heated and then allowed to cool, the same amount of heat will always be lost whilst it cools over the range of temperature of 5° F. marked on the stem, but the rate of heat loss will depend upon the atmospheric conditions. It has, in fact, been found that if the kata-thermometer is exposed to a current of air moving with speed V, the mean rate of cooling over the temperature range, if the bulb is quite dry, is related to the air speed. The law takes the form

$$H = (a + b\sqrt{V})\theta, \qquad . \quad . \quad . \quad (16)$$

where $a$ and $b$ are constants,

- $\theta$ is the mean excess temperature of the kata-thermometer over the surrounding air, and
- H is the total heat lost in cooling from 100° F. to 95° F., divided by the surface area of the bulb in sq. cms., and by the cooling time.

It will be noted that this law is of the same form as that found by King for heated wires.

The total heat lost in cooling over a given range of temperature, although constant for a given instrument, will vary from one kata-thermometer to another. Despite the fact that it is endeavoured to make all instruments to a given size, slight variations in heat capacity still occur between different instruments. Each kata-thermometer is therefore calibrated in still air by the makers, and is given a " factor " F, which is the total heat lost in milli-calories per square centimetre of bulb surface, in cooling from 100° F. to 95° F. In order, therefore, to calculate H in equation (16), it is only necessary to observe the time T in seconds taken by the alcohol column to fall from the 100° mark to the 95° mark, and to divide the factor F, which is always stated on each instrument, by this time.

From a large number of experiments conducted with different kata-thermometers, the constants $a$ and $b$ in equation (16) have been determined. When V is expressed in feet per minute, and temperatures are measured on the Fahrenheit scale, this equation becomes

$$\frac{F}{T} = (0{\cdot}11 + 0{\cdot}016\sqrt{V})(97{\cdot}7 - t) \quad . \quad . \quad (17)$$

for values of V less than 180 feet per minute,

and $$\frac{F}{T} = (0{\cdot}07 + 0{\cdot}019\sqrt{V})(97{\cdot}7 - t) \quad . \quad . \quad (18)$$

for values of V above 180 feet per minute.

In these equations T is the observed time in seconds for the alcohol to fall through the standard temperature range, and $t$ is the atmospheric temperature in degrees Fahrenheit.

Although these relationships are supposed to be general for all kata-thermometers, until the results of further work have been published upon the use of these instruments in anemometry, it would seem to be desirable, for the best accuracy, to calibrate each instrument separately. The calibration can be easily performed by the method mentioned in connection with the constant resistance hot-wire anemometer. Thus, $a$ in equation (16) is given by the value of $\frac{F}{T}$ in still air when $V = 0$, and $b$ can then be determined by a calibration at a single known air speed, although it is preferable to estimate it for a number of speeds and to take a mean value.

The procedure in using this instrument as an anemometer is as follows: It is convenient, if a number of observations have to be made, to start work with a vacuum flask filled with boiling water, in order to provide a ready means of heating the kata-thermometer to a temperature of over 100° F. The bulb of the kata-thermometer is placed in the water and is allowed to remain there until an unbroken column of alcohol fills the stem and about half the small bulb at the top of the stem. The instrument is then quickly removed from the water, carefully dried—this point is important—and then introduced into the airstream to be measured. When in this position, the kata-thermometer must be fixed to prevent swaying, and the bulb must be freely exposed to the current. An observation of the time of cooling over the standard temperature range is then made. The inventor states that this process should be repeated four or six times, according to the agreement obtained between the different observations. It is frequently found that the first reading differs quite widely from the rest, and it should always be neglected, the mean value of the cooling time being calculated from the remaining observations. The temperature of the air is observed during the tests, and equation (17) or (18) can then be used to determine V.

Approximate corrections for variations of air density may be applied on the assumption that, as in the case of the hot-wire anemometer, the heat loss is dependent upon $\sqrt{V\sigma}$, instead of $\sqrt{V}$ only, so that equation (16) may be written

$$\frac{F}{T} = (a + b\sqrt{V\sigma})\theta,$$

where $a$ and $b$ have the appropriate values given above.

The method of making these corrections has already been indicated in the discussion of the hot-wire anemometer.

# BIBLIOGRAPHY.

## Theoretical.


Boussinesq. *Comptes Rendus*, vol. 133, p. 257, and *Journal de Mathématiques*, vol. 1, p. 285. Theoretical discussion of heat losses from bodies of various shapes immersed in a stream of fluid.

Russell. *Phil. Mag.*, vol. 20, 1910, p. 591. An account of the theoretical development of the subject, with numerous references.

L. V. King. *Phil. Trans. Roy. Soc.*, vol. 214, 1914, p. 373. Approximate solutions of Boussinesq's equations are derived for cylinders of small $\frac{\text{diameter}}{\text{length}}$ ratio.

Davis. *Phil. Mag.*, vol. 40, 1920, p. 692; vol. 41, 1921, p. 899; and vol. 43, 1921, p. 239. A discussion of the subject from the standpoint of similarity.


## Experimental—Measurements of Heat Losses from Heated Wires.


Ayrton and Kilgour. *Phil. Trans. Roy. Soc.*, vol. 183, part 1, 1892, p. 371.

Petavel. *Phil. Trans. Roy. Soc.*, vol. 191, 1898, p. 501; also vol. 197, 1901, p. 229.

Kennelly, Wright, and Van Bylevelt. *Trans. Amer. Inst. Elect. Eng.*, vol. 26, 1907, p. 969; also vol. 28, 1909, p. 363.

Langmuir. *Phys. Rev.*, vol. 34, 1912, p. 401; also *Proc. Amer. Inst. Elect. Eng.*, June, 1912, p. 1011, and April, 1913.

L. V. King. *Phil. Trans. Roy. Soc.*, vol. 214, 1914, p. 373; also *Phil. Mag.*, vol. 29, 1915, p. 556; also *Journ. Franklin Inst.*, vol. 181, 1916, p. 1, and vol. 183, 1917, p. 783.


## Hot-Wire Anemometry.


Bordini. *Nuovo Cimento*, Series VI., vol. 3, 1912, p. 241; see also *The Electrician*, vol. 70, 1912, p. 278.

Morris. *The Engineer*, Sept. 27, 1912; also *The Electrician*, Oct. 4, 1912, p. 1056.

Gerdien. *Ber. der Deutschen Phys. Ges.*, vol. 20, 1913.

L. V. King. *Loc. cit.*

Fage. Reports and Memoranda of the Aeronautical Research Committee, No. 556, Nov., 1918.

R. O. King. *Engineering*, vol. 117, 1924, pp. 136 and 249.

Davis. *Proc. Phys. Soc., Lond.*, vol. 33, 1921, p. 152.

Simmons and Bailey. *Phil. Mag.*, vol. 3, 1927, p. 81.

### Electric Flow Meters.

C. C. Thomas. *Journ. Franklin Inst.*, vol. 172, 1911, p. 411.

Moss and Stern. Aeronautical Research Committee—Engine Sub-Committee Reports, No. 55, Sept., 1920.

### The Kata-Thermometer.

Hill, Griffith, and Flack. *Phil. Trans. Roy. Soc.*, B., vol. 207, 1916, p. 186.

Hill, Vernon, and Hurgood-Ash. *Proc. Roy. Soc. Med.*, B., vol. 93, 1922, p. 198.

Angus. *Journ. Industrial Hygiene*, vol. 6, 1924, p. 2.

See also Medical Research Council, Special Report Series, Nos. 32 (1919), 52 (1920), and 73 (1923).

# INDEX.

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